HOLES IN POSSIBLE VALUES OF h^1 AND GEOMETRIC GENUS.

JÁNOS NAGY

Abstract.

In the theory of normal surface singularities in the last decade one of the major issues was to compare it's analytic invariants with it's topological invariants, which are computable just from the resolution graph $\mathcal{T}$ of the given singularity, or equivalently from the link of the singularity, which is an oriented graph 3-manifold.

The main subject of research was to provide topological formulae for several discrete analytic invariants or at least topological candidates. Of course, when we fix the topological type and vary the analytic structure most of these analytic invariants also can change, so we have a hope to find purely topological formulas just in the case of special analytical families.

One such large breakthrough was the previous work of Némethi, and Okuma, that for splice quotient singularities, or in an another language for singularities satisfying the end curve condition, many analytic invariants like geometric genus or analytic Poincaré series coincide with their topological candidates like Seiberg-Witten invariants and topological multivariable Zeta function.

The coincidence of the Seiberg-Witten invariant and the geometric genus also holds in the case of Newton nondegenerate hypersurface singularities.

There is an another topological candidate which is called path lattice cohomology, or in other articles MIN_γ, which is also an upper bound for the geometric genus for every analytic structure, however for some special families of singularities there is equality, like in the Superisolated case or also in the case of Newton nondegenerate hypersurface singularities .

However there are resolution graphs, for which MIN_γ is not the geometric genus of any singularity corresponding to it, so the upper bound is not sharp in general.

While the minimal possible geometric genus for a fixed topological type is already known, which is the geometric genus of generic singularities, the determination of the maximal possible geometric genus is still an open problem.

The main massage of these results is that while the geometric genus can change when we vary the analytic structure, there are combinatorial candidates for this value, and equality happens for special families of analytic types.

However we show, that if we take all possible analytic structures into consideration, then the possible values of the geometric genus $p_g(\widetilde{X})$ form an interval of integers.

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