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Title: Categorification with Lattice Homology

Abstract: T. Ágoston and A. Némethi initiated recently the study of lattice homology theories associated with ‘split’ weight functions, obtained as the difference of two ‘height functions’. In this talk we will be considering the case when the height functions come as Hilbert functions of some valutive multifiltrations on a Noetherian k -algebra O and a finitely generated module M over it. We introduce the notion of ‘realizable submodules’ in M , the prime example of which are finite codimensional integrally closed submodules. Our Independence Theorem states, that whenever two sets of (extended) discrete valuations ‘realize’ the same submodule N in M , then the resulting lattice homology is the same, in fact, it is a well-defined invariant of the quotient M/N . Moreover, it has Euler characteristic $\dim(M/N)$.

The main output of the Independence Theorem is this possibility of categorifying numerical invariants defined as codimensions of realizable submodules: e.g., the delta invariant of reduced curve singularities; or the geometric genus, the irregularity and the various plurigenera of higher dimensional isolated normal singularities. We will also discuss structural bounds, symmetry properties and relations with deformation theory.

The results come from ongoing joint work with A. Némethi.