ADDITIVE COMBINATORICS

Reference Text: **T. Tao, V. Vu**, Additive Combinatorics, Cambridge University Press, 2006 or **M. B. Nathanson**, Additive Number Theory, Inverse Problems and the Geometry of Sumsets, Springer Verlag, 1996.

Course description: Our aim is to give an introduction to Additive Combinatorics, one of the most recent and most dynamically developing branch of Number Theory. We will cover classical direct problems, such as Roth's Theorem about three term arithmetic progressions in dense sets, or sum–product estimates, as well as inverse problems, such as Sidon sets, or Freimann's Theorem about the classification of sets with small doubling. The most spectacular results of the subject, Szemerédi's Theorem about long arithmetic progressions in dense sets or Green–Tao's Theorem about long arithmetic progressions in primes are beyond the scope of a one semester course. However, we will present Gowers' proof for Szemerédi's Theorem in the case of four term arithmetic progressions, as well as we give a glimpse to the general case. The interested students get the necessary basis to continue their studies in this interesting field.

Prerequisites: A first course of number theory; you must be familiar with prime numbers, divisibility and congruences. Algebra; you must be familiar with the basic concepts of linear algebra, groups, and finite fields. Calculus; you must be familiar with functions, differential calculus, integrals. Combinatorics; you must be familiar with the basic concept of graphs, but no graph theoretic results are used in this course. Actually the course uses rather elementary tools with very little prerequisite, still the arguments are going to be deep and involved, sometimes very complex and long.

REVIEW of basic Number Theory

You must be familiar with the following subjects. Here you find the reference to the two handbooks, [Z.3.2] means Chapter 3.2 of I. Niven, H. S. Zuckerman, H. L. Montgomery, An Introduction to the Theory of Numbers, while [N.3.5] means Chapter 3.5 of M. B. Nathanson, Elementary Methods in Number Theory.

– Divisibility, [Z.1.2] or [N.1.1–1.3].

– Fundamental Theorem of Arithmetic, [Z.1.3] or [N.1.4–1.5].

– Congruences, [Z.2.1–2.2] or [N.2.1–2.2, 2.5].

– Chinese Remainder Theorem, [Z.2.3] or [N.2.3–2.4].

– Congruences of higher degree, [Z.2.6–2.7] or [N.3.6].

– Primitive roots and power residues, [Z.2.8] or [N.3.1–3.3].

– Quadratic residues, [Z.2.9, 3.1] or [N.3.4].

– Quadratic reciprocity, [Z.3.2] or [N.3.5].