INTRODUCTION TO ANALYTIC NUMBER THEORY

Reference Text: H. Davenport, Multiplicative Number Theory, M. B. Nathanson, Elementary Methods in Number Theory, I. Niven, H. S. Zuckerman, H. L. Montgomery, An Introduction to the Theory of Numbers, J. B. Conway, Functions of one complex variable and L. V. Ahlfors, Complex Analysis. All books are around, you can find or borrow them (in limited amount) at the students coordinators office.

Subject: Our aim is to provide a classical introduction to analytic number theory, focused on the connection between the zeros of the Riemann ζ–function and prime numbers. The highlights are a proof of the Prime Number Theorem and of Dirichlet's Theorem about the distribution of primes in arithmetic progressions but the course is much more than just the proof of two famous theorems.

Prerequisites: A first course of number theory; you must be familiar with prime numbers, divisibility, congruences, quadratic residues, primitive roots. Calculus; you must be familiar with differential calculus, Taylor series, Fourier series, integrals. Complex analysis; you must be familiar with the arithmetic of complex numbers, differential calculus of complex functions of a complex variable, contour integrals, analytic functions and Taylor series. Taking CLX parallel to this course is more than enough. My personal opinion about a little lack of knowledge is that a good level of calculus is essential. If you are familiar with complex numbers and you plan to take a CLX type course (now or) later in your studies, you can accept what we do. If you are familiar with divisibility and congruences you can make up quickly the rest of basic number theory.

REVIEW of basic Number Theory

You must be familiar with the following subjects. Here you find the reference to the two handbooks, [Z.3.2] means Chapter 3.2 of I. Niven, H. S. Zuckerman, H. L. Montgomery, An Introduction to the Theory of Numbers, while [N.3.5] means Chapter 3.5 of M. B. Nathanson, Elementary Methods in Number Theory.

– Divisibility, [Z.1.2] or [N.1.1–1.3].

– Fundamental Theorem of Arithmetic, [Z.1.3] or [N.1.4–1.5].

– Congruences, [Z.2.1–2.2] or [N.2.1–2.2, 2.5].

– Chinese Remainder Theorem, [Z.2.3] or [N.2.3–2.4].

– Congruences of higher degree, [Z.2.6–2.7] or [N.3.6].

– Primitive roots and power residues, [Z.2.8] or [N.3.1–3.3].

– Quadratic residues, [Z.2.9, 3.1] or [N.3.4].

– Quadratic reciprocity, [Z.3.2] or [N.3.5].

REVIEW of basic complex function theory

You must be familiar with the following subjects. Here you find the reference to the two handbooks, [A.2.3.2] means Chapter 2, Section 3.2 of L. V. Ahlfors, Complex Analysis, while [C.3.1] refers to Chapter 3, Section 1 of J. B. Conway, Functions of one complex variable.

– The algebra of complex numbers, [A.1.1.1–1.2.2], [C.1.2-1.5].

– Complex functions, [A.2.1.1–2.3.4], [C.3.1-3.3, 4.2-4.3].

– Complex integration, [A.4.1.1–4.1.5, 4.4.4], [C.4.1].

– Cauchy's integral formula, [A.4.2.1–4.2.3], [C.4.5].

– Uniqueness theorem, [A.4.3.2], [C.4.3].

– The maximum principle, [A.4.3.3–4.3.4], [C.6.1].

– The residue theorem, [A.4.5.1], [C.5.1-5.3].

– Taylor series, [A.5.1.1–5.1.2], [C.3.1].