The code for semidefinite optimization

In[10]:=
```
QOT[ρ_, ω_, cost_] := Module[{σ, A, m, C, c, sol, Π},
σ = {{{1, 0}, {0, 1}}, {{0, 1}, {1, 0}}, {{0, -I}, {I, 0}}, {{1, 0}, {0, -1}}};
A = Flatten[Table[KroneckerProduct[σ[[j]], σ[[k]]], {j, 1, 4}, {k, 1, 4}], 1];
m = Length[cost];
C = Sum[MatrixPower[KroneckerProduct[cost[[l]], IdentityMatrix[2]] -
       KroneckerProduct[IdentityMatrix[2], Transpose[cost[[l]]]], 2], {l, 1, m}];
c = Table[Tr[A[[n]].C], {n, 1, 16}];
sol = SemidefiniteOptimization[
     c.{1/4, x12, x13, x14, x21, x22, x23, x24, x31, x32, x33, x34, x41, x42, x43, x44},
{1/4 σ[[1]] + x21 σ[[2]] + x31 σ[[3]] + x41 σ[[4]] == ω/2,
      1/4 σ[[1]] + x12 σ[[2]] + x13 σ[[3]] + x14 σ[[4]] == Transpose[ρ]/2,
 VectorGreaterEqual[{{1/4, x12, x13, x14, x21, x22, x23, x24, x31, x32,
           x33, x34, x41, x42, x43, x44}.A, 0}, {"SemidefiniteCone", 4}]},
{x12, x13, x14, x21, x22, x23, x24, x31, x32, x33, x34, x41, x42, x43, x44}];
Π = Chop[A[[1]]/4 + Sum[sol[[n, 2]] A[[n + 1]], {n, 1, 15}]];
{Sqrt[Re[Tr[C.Π]]], MatrixForm[Π]}
]
```

Numerics on the "clock and shift" cost operator

In[11]:=
```
P[x_, y_, z_] := 1/2 {{1 + z, x - I y}, {x + I y, 1 - z}}
```

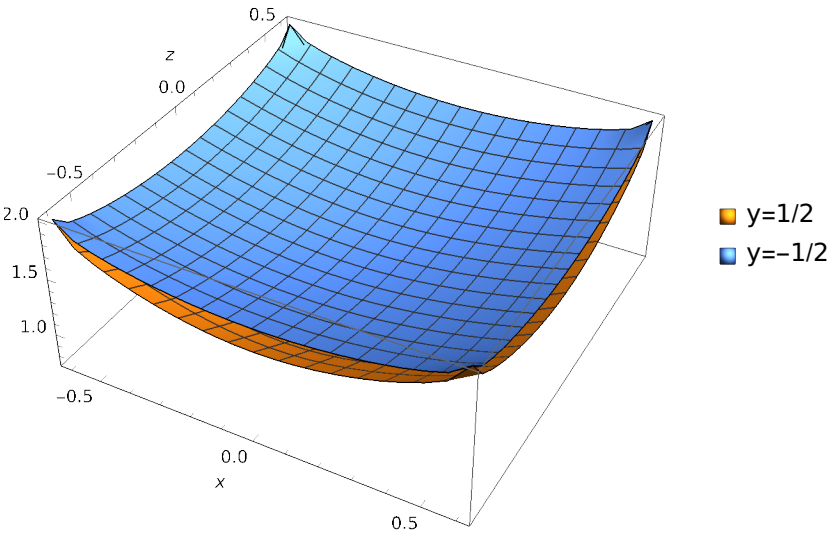
In[12]:=
```
C13 = {{{0, 1}, {1, 0}}, {{1, 0}, {0, -1}}};
```

Case 1: y=+-1/2

In[13]:=
```
T1 = {Table[QOT[P[x Sqrt[3/8], 1/2, z Sqrt[3/8]], P[0, 1/2, 0], C13][[1]],
     {x, -1, 1, 0.1}, {z, -1, 1, 0.1}],
Table[QOT[P[x Sqrt[3/8], -1/2, z Sqrt[3/8]], P[0, 1/2, 0], C13][[1]],
     {x, -1, 1, 0.1}, {z, -1, 1, 0.1}]};
```

In[14]:=
```
ListPlot3D[
T1,
DataRange → {{-Sqrt[3/8], Sqrt[3/8]}, {-Sqrt[3/8], Sqrt[3/8]}},
PlotLegends → {"y=1/2", "y=-1/2"},
AxesLabel → {x, z}
]
```

Out[14]=

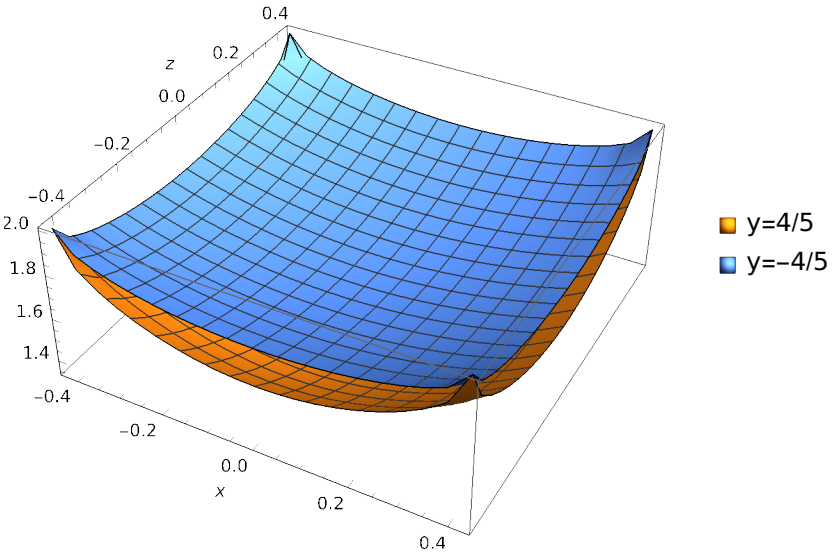


Case 2: y=+-4/5

In[15]:=
```
T2 = {Table[QOT[P[x Sqrt[9/2]/5, 4/5, z Sqrt[9/2]/5], P[0, 1/2, 0], C13][[1]],
    {x, -1, 1, 0.1}, {z, -1, 1, 0.1}],
Table[QOT[P[x Sqrt[9/2]/5, -4/5, z Sqrt[9/2]/5], P[0, 1/2, 0], C13][[1]],
    {x, -1, 1, 0.1}, {z, -1, 1, 0.1}]};
```

In[16]:=
```
ListPlot3D[
T2,
DataRange → {{- Sqrt[9 / 2] / 5, Sqrt[9 / 2] / 5}, {- Sqrt[9 / 2] / 5, Sqrt[9 / 2] / 5}},
PlotLegends → {"y=4/5", "y=-4/5"},
AxesLabel → {x, z}
]
```

Out[16]=

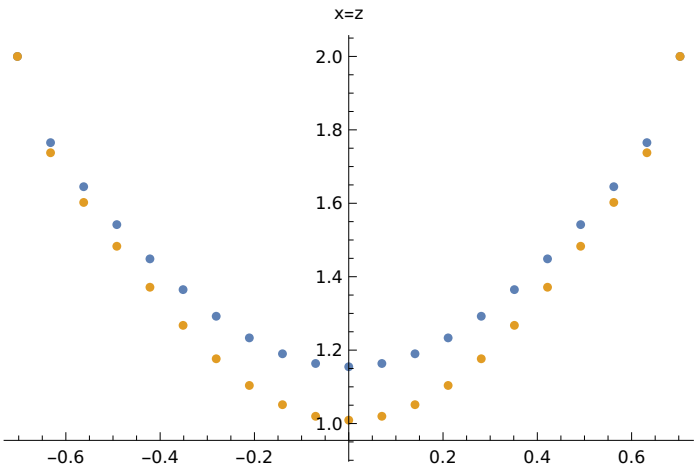


Case 3: y=+-1/9

In[19]:=
```
T3 =
  {Table[QOT[P[t Sqrt[40 / 81], -1 / 9, t Sqrt[40 / 81]], P[0, 1 / 2, 0], C13][[1]], {t, -1, 1, 0.1}],
Table[QOT[P[t Sqrt[40 / 81], 1 / 9, t Sqrt[40 / 81]], P[0, 1 / 2, 0], C13][[1]], {t, -1, 1, 0.1}]};
```

In[20]:=
```
ListPlot[
T3,
DataRange → {- Sqrt[40 / 81], Sqrt[40 / 81]},
AxesLabel → "x=z"
]
```

Out[20]=



ListPlot