```
In[∘]:= RandSelfadjMatrix[l_] := Module[{A, i},
    A = RandomVariate[NormalDistribution[], {l, l}] +
      I * RandomVariate[NormalDistribution[], {l, l}];
    A = A + ConjugateTranspose[A];
    A = Chop[A];
    A]

In[∘]:= RandPositive[l_, r_] := Module[{A},
    A = RandomVariate[NormalDistribution[], {l, r}] +
      I * RandomVariate[NormalDistribution[], {l, r}];
    A = A.ConjugateTranspose[A];
    A]

In[∘]:= RandState[l_, r_] := Module[{A},
    A = RandPositive[l, r];
    A = A / Tr[A];
    A]

In[∘]:= CostfromObservable[Observables_, transpose_] := Sum[
    MatrixPower[KroneckerProduct[Observables⟦k⟧, IdentityMatrix[Length[Observables⟦1⟧]]] -
      KroneckerProduct[IdentityMatrix[Length[Observables⟦1⟧]], If[transpose,
        Transpose[Observables⟦k⟧], Observables⟦k⟧]], 2], {k, 1, Length[Observables]}]

In[∘]:= SaMCNB[dim_] :=
    SaMCNB[dim] = Flatten[{{Table[SparseArray[{{k, k} → 1, {dim, dim} → 0}], {k, 1, dim}]},
       Table[Table[SparseArray[{{k, m} → Sqrt[2] / 2, {m, k} → Sqrt[2] / 2, {dim, dim} → 0}],
         {m, k + 1, dim}], {k, 1, dim - 1}], Table[Table[SparseArray[{{k, m} → -I * Sqrt[2] / 2,
           {m, k} → I * Sqrt[2] / 2, {dim, dim} → 0}], {m, k + 1, dim}], {k, 1, dim - 1}]}, 2];

In[∘]:= SaMPB[2] = {{{1, 0}, {0, 1}}, {{0, 1}, {1, 0}}, {{0, -I}, {I, 0}}, {{1, 0}, {0, -1}}};
SaMPB[dim_] := SaMPB[dim] = Flatten[Table[
      KroneckerProduct[SaMPB[dim / 2]⟦j⟧, SaMPB[2]⟦k⟧], {j, 1, dim^2 / 4}, {k, 1, 4}], 1];

In[∘]:= SaMPNB[dim_] := SaMPNB[dim] = SaMPB[dim] / Sqrt[dim];
```

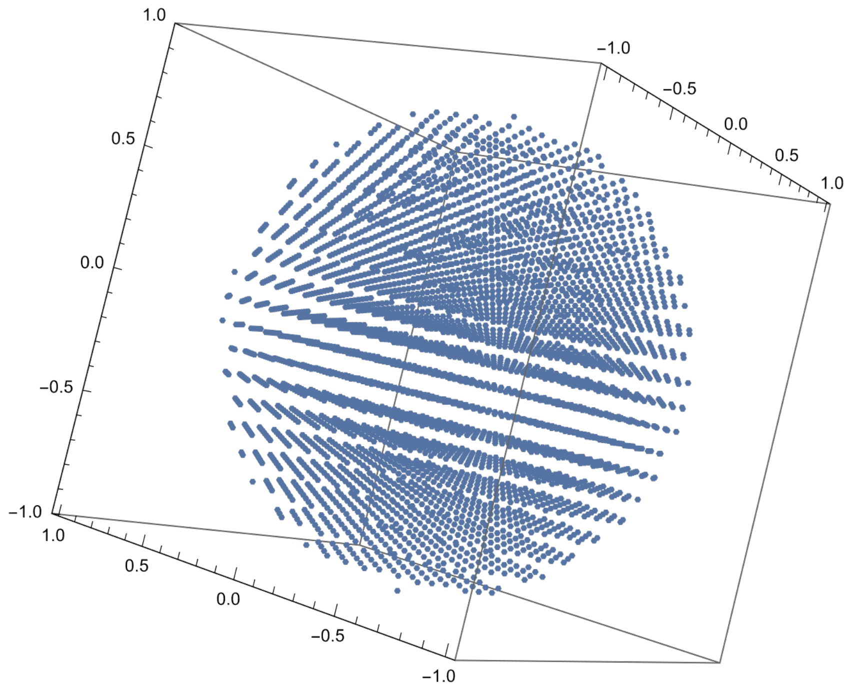
```
In[•]:= QOT[ρ_, ω_, C_, dual_, transpose_] := Module[{dim, sol, Π, x, y, X, Y},
          dim = Length[ρ];
          If[
           dual, {x = Table[Symbol["x" <> ToString[n]], {n, dim^2}];
            y = Table[Symbol["y" <> ToString[n]], {n, dim^2}];
            sol = SemidefiniteOptimization[-Tr[(y.SaMCNB[dim]).ω + (x.SaMCNB[dim]).ρ],
              {VectorGreaterEqual[{C - KroneckerProduct[(y.SaMCNB[dim]), IdentityMatrix[dim]] -
                  KroneckerProduct[IdentityMatrix[dim], If[transpose, Transpose[
                     (x.SaMCNB[dim])], x.SaMCNB[dim]]], 0}, {"SemidefiniteCone", dim^2}]} ,
        Flatten[{x, y}]];
            X = Chop[sol[[1 ;; dim^2, 2]].SaMCNB[dim], 10^-3];
            Y = Chop[sol[[dim^2 + 1 ;; 2 * dim^2, 2]].SaMCNB[dim], 10^-3];
        Chop[Sqrt[Tr[X.ω + Y.ρ]], 10^-3], X // MatrixForm, Y // MatrixForm},

           {x = Table[Symbol["x" <> ToString[n]], {n, dim^4}];
            sol = SemidefiniteOptimization[Chop[Simplify[Tr[(x.SaMCNB[dim^2]).C]], 10^-3],
        {ResourceFunction["MatrixPartialTrace"][x.SaMCNB[dim^2], 2, {dim, dim}] == ω,
               ResourceFunction["MatrixPartialTrace"][x.SaMCNB[dim^2], 1, {dim, dim}] ==
                If[transpose, Transpose[ρ], ρ],
         VectorGreaterEqual[{x.SaMCNB[dim^2], 0}, {"SemidefiniteCone", dim^2}] },
        x];
        Π = Chop[Sum[sol[[n, 2]] SaMCNB[dim^2][[n]], {n, 1, dim^4}], 10^-3];
        Chop[Sqrt[Tr[C.Π]], 10^-3], Π // MatrixForm}]]

In[•]:= ModQOT [ρ_, ω_, cost_, dual_, transpose_] := Sqrt[QOT[ρ, ω, cost, dual, transpose][[1]]^2 -
          (QOT[ρ, ρ, cost, dual, transpose][[1]]^2 + QOT[ω, ω, cost, dual, transpose][[1]]^2) / 2]

In[•]:= TriIneq[ρ_, ω_, τ_, C_, dual_, transpose_] := ModQOT[ρ, ω, C, dual, transpose] +
          ModQOT[ω, τ, C, dual, transpose] - ModQOT[ρ, τ, C, dual, transpose]
```

```
In[•]:= Cost1 = CostfromObservable[
          {RandSelfadjMatrix[2], RandSelfadjMatrix[2], RandSelfadjMatrix[2]}, True];

In[•]:= Cost2 = CostfromObservable[
          {RandSelfadjMatrix[2], RandSelfadjMatrix[2], RandSelfadjMatrix[2]}, True];

In[•]:= Cost3 = CostfromObservable[{RandSelfadjMatrix[2],
           RandSelfadjMatrix[2], RandSelfadjMatrix[2], RandSelfadjMatrix[2]}, True];

In[•]:= Cost4 = CostfromObservable[{RandSelfadjMatrix[2],
           RandSelfadjMatrix[2], RandSelfadjMatrix[2], RandSelfadjMatrix[2]}, True];

In[•]:= ρ01 = RandState[2, 2]; τ01 = RandState[2, 2];

In[•]:= ρ02 = RandState[2, 2]; τ02 = RandState[2, 2];

In[•]:= ρ03 = RandState[2, 2]; τ03 = RandState[2, 2];

In[•]:= ρ04 = RandState[2, 2]; τ04 = RandState[2, 2];

In[•]:= step = 0.1;
```

In[∘]:=
```
coordinates =
  Select[Flatten[Table[{x, y, z}, {x, -1, 1, step}, {y, -1, 1, step}, {z, -1, 1, step}], 2],
   PositiveSemidefiniteMatrixQ[1 / 2 * (Flatten[{1, #}]).SaMPB[2]] &];
```

In[∘]:=
```
ListPointPlot3D[coordinates, BoxRatios → {1, 1, 1}]
```

Out[∘]=



In[∘]:=
```
ω2dim = 1 / 2 * (Flatten[{1, #}]).SaMPB[2] & /@ coordinates;
```

In[∘]:=
```
Off[SemidefiniteOptimization::parsuc]
```

```
In[•]:= ToExpression[ToString[Flatten[Table[{"time" <> ToString[i] <> ToString[j],
            "result" <> ToString[i] <> ToString[j]}, {i, 4}, {j, 4}], 1]] <>
        "=Flatten[Table[AbsoluteTiming[TriIneq[ToExpression[\"ρ0\"<>ToString[i]],#,
          ToExpression[\"τ0\"<>ToString[i]],ToExpression[\"Cost\"<>ToString[j]],False,True]
          &/@ω2dim],{i,4},{j,4}],1];"]

In[•]:= Join[{{"time", "result"}}, ToExpression[
        ToString[Flatten[Table[{"time" <> ToString[i] <> ToString[j], "Min[result" <>
              ToString[i] <> ToString[j] <> "]"}, {i, 4}, {j, 4}], 1]]]] // MatrixForm
```

Out[•]//MatrixForm=

time	result
678.74	0.310819
618.762	0.528506
571.357	0.760247
528.565	0.352543
507.574	0.218016
492.109	0.715538
497.975	0.590063
496.675	0.453942
496.888	0.280697
496.107	0.642319
530.554	0.669042
676.222	0.800527
617.609	0.195821
654.952	0.44385
752.794	0.447589
638.314	0.401331

```
In[•]:= {dim3time, dim3result} =
         AbsoluteTiming[Table[TriIneq[RandState[3, 3], RandState[3, 3], RandState[3, 3],
            CostfromObservable[{RandSelfadjMatrix[3], RandSelfadjMatrix[3],
              RandSelfadjMatrix[3]}, True], False, True], 4000]];

In[•]:= dim3time / 3600

Out[•]= 0.842184

In[•]:= Min[dim3result]

Out[•]= 0.854168

In[•]:= {dim4time, dim4result} =
         AbsoluteTiming[Table[TriIneq[RandState[4, 4], RandState[4, 4], RandState[4, 4],
            CostfromObservable[{RandSelfadjMatrix[4], RandSelfadjMatrix[4],
              RandSelfadjMatrix[4]}, True], False, True], 4000]];

In[•]:= dim4time / 3600

Out[•]= 7.36725
```

In[ ]:=
```
Min[dim4result]
```

Out[ ]=
```
1.89892
```

In[ ]:=
```
{dim5time, dim5result} =
  AbsoluteTiming[Table[TriIneq[RandState[5, 5], RandState[5, 5], RandState[5, 5],
     CostfromObservable[{RandSelfadjMatrix[5], RandSelfadjMatrix[5],
       RandSelfadjMatrix[5]}, True], False, True], 4000]];
```

In[ ]:=
```
dim5time / 3600
```

Out[ ]=
```
67.7145
```

In[ ]:=
```
Min[dim5result]
```

Out[ ]=
```
2.69551
```