INTRODUCTION TO ASYMPTOTIC EXPANSIONS

Course coordinator: Gergő Nemes

No. of Credits: 3, and no. of ECTS credits: 6

Prerequisites: Complex Function Theory

Course Level: introductory PhD

Brief introduction to the course:
We discuss the classical methods of the asymptotic theory of integrals like the integration by parts, Watson's lemma, Laplace's method, the principle of stationary phase and the method of steepest descents.

The goals of the course:
The aim of the course is to introduce the students to the classical theory of asymptotic power series.

The learning outcomes of the course:
By the end of the course, students become familiar with the classical methods of asymptotic analysis, and they are able to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and science, in general.

More detailed display of contents (week-by-week):

Week 1: Asymptotic notations, asymptotic sequences and expansions, failure of uniqueness, asymptotic sum, uniform asymptotic expansions

Week 2: Asymptotic power series, basic operations on asymptotic power series, integration and differentiation, relation to Laurent series, Love's theorem

Week 3: Incomplete gamma functions, the method of integration by parts, error bounds, the first encounter with the Stokes phenomenon

Week 4: Watson's lemma for real integrals, the asymptotic expansion of the modified Bessel function for large argument, Digamma function, the asymptotic expansion of the logarithm of the Gamma function

Week 5: Laplace's approximation, Stirling's formula, the asymptotics of the Legendre polynomials for large order, further examples

Week 6: Laplace's method, the asymptotic expansion of the Gamma function, Stirling coefficients, modified Bessel function of large order and argument

Week 7: The principle of stationary phase, the asymptotic behaviour of the Airy function, Bessel functions of large order and argument

Week 8: Watson's lemma for complex integrals, the asymptotic expansion of the Incomplete gamma function, the method of steepest descents

Week 9: Applications of the method of steepest descents: the Gamma function revisited, asymptotic expansions for the Airy function, Stokes' phenomenon

Week 10: Debye's expansions for the Bessel function

Week 11: The saddle point method, asymptotic approximation for the coefficients in Debye's third expansion, the asymptotics of the Legendre polynomials for large order

Week 12: Brief introduction to exponential asymptotics, optimal truncation, Ursell's lemma, asymptotic approximations for the remainders

Reference:

N. Bleistein, R. A. Handelsman, *Asymptotic Expansion of Integrals*, Holt Rinehart and Winston, New York, 1975.

N. G. de Bruijn, *Asymptotic Methods in Analysis*, Amsterdam, North-Holland; Groningen, Noordhoff; New York, Interscience, 1958.

E. T. Copson, *Asymptotic Expansions*, Cambridge University Press, 1965.

A. Erdelyi, *Asymptotic Expansions*, Dover, New York, 1956.

J. D. Murray, *Asymptotic Analysis*, Springer, New York, 1984.

F. W. J. Olver, *Asymptotics and Special Functions*, A. K. Peters Ltd., Wellesley, 1997.

R. Wong, *Asymptotic Approximations of Integrals*, Boston–New York: Academic Press Inc. Reprinted with corrections by SIAM, Philadelphia, PA, 2001.

Assessment:
Grading is based on homeworks that will be assigned regularly. Students registered for audit should attend at least 80% of the lectures to pass.