

Fast transformations of Latin squares, half-regular factorizations and edge colorings of bipartite graphs in small steps

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Abstract

Jacobson and Matthew gave perturbations that are necessary and sufficient to transform any $n \times n$ Latin square to any another $n \times n$ Latin square. In some sense, these perturbations are small: a perturbation changes at most 3 rows. In some sense, they are large: unlimited number of columns might be affected by a single perturbation. These perturbations can be used in a Markov chain Monte Carlo framework to generate random Latin squares. Based on certain properties of the so-emerging Markov chain, these perturbations can be considered as small ones. They also can be considered as fast in the sense that a small number of such perturbations are sufficient to transform any Latin square into any another one.

We show that similar perturbations are possible for half-regular factorizations of complete bipartite graphs and edge colorings of bipartite graphs. That is, we give perturbations that are necessary and sufficient to transform any half-regular factorization of a complete bipartite graph into another half-regular factorization of the same complete bipartite graph. A perturbation affects at most 3 vertices in the first (regular) vertex class and unlimited number of vertices in the second vertex class. These perturbations can be applied in a Markov chain Monte Carlo framework to generate random factorizations, and can be considered as small perturbations. Furthermore, they can be considered as fast because a small number of perturbations are sufficient to transform any factorization into any another one.

Similarly, we give perturbations that are necessary and sufficient to transform any edge k -coloring to another edge k -coloring of a bipartite graph with maximum degree $\Delta \leq k$. A perturbation affects at most 3 colors and unlimited number of edges. They can be applied in a Markov chain Monte Carlo framework to generate random edge colorings, and they can be considered as small perturbations. Also, they are fast: a small number of perturbations are sufficient to transform any edge k -colorings into another one. A special case is when the bipartite graph is k -regular, and it is easy to see that this case is equivalent with the completion of Latin rectangles.

Joint work with former BSM students Mark Aksen, Kathleen Zhou and Letong Hong.

References

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