Combinatorics and Graph Theory II

Practice for Midterm 1, October 9

You have 90 minutes for the problem set. Please write your **name** and **neptun code** on each sheet. Each problem is worth 10 points. Results without proper argument are not marked. Partial results get partial points. You need to get 24 points to pass.

Pen, paper and problem sets 1-5 can be used, nothing else.

Good luck!

1. Give all plane graphs G (graphs drawn in the plane with no crossings) with $e(G) \leq f(G)$.

2. G is a graph of 100 vertices. We know that if we remove ANY edge of G, we get a planar graph. Prove that $\alpha(G) \geq 24$. ($\alpha(G)$ is the independence number of G)

3. G is a simple plane graph with 2023 edges. Prove that it is not isomorphic to its dual.

4. Let $n > 0$, the vertices of G correspond to n different points in the plane. Two vertices are connected, if and only if the corresponding points have distance at least 1. Is G perfect for every set of n points?

5. Determine the list chromatic number $ch(C_n)$ of C_n, the cycle of length n, for every $n \geq 3$.

6. Let $n > 0$ and let S be an n-element set. The vertices of G correspond to the subsets of S (se G has 2^n vertices) and two vertices are connected if and only if one of the corresponding subsets contains the other. Show that G is perfect.