Combinatorics and Graph Theory II

Practice for Midterm 2, November 22

You have 90 minutes for the problem set. Please write your **name** and **neptun code** on each sheet. Each problem is worth 10 points. Results without proper argument are not marked. Partial results get partial points. You need to get 24 points to pass.

Pen, paper and problem sets 1-5 can be used, nothing else.

Good luck!

1. Construct a graph G with $ch(G) = 100$ and $\Delta(G) = 2023$.

2. Prove that $R(3, 3, 3, 3, 3, 2) \leq R(6, 18)$

3. G is a 7-vertex graph, the disjoint union of a K_5 and an edge. Determine $ex(G, 100)$.

4. Prove that for any r there is an $N(r)$ with the following property. If we color the numbers $1, 2, \dots, N(r)$ with r colors, then there are a, b, c of the same color such that $a + b = c + 1$.

5. Suppose that $\mathcal{F} \subseteq 2^{[2023]}$ and there are no three sets in it such that $A \subset B \subset C$. What is the maximum of $|\mathcal{F}|$?

6. G is a bipartite graph, its vertex classes are A and B, $|A| = 25$, $|B| = 20$. Any two vertices in A have exactly two common neighbors in B.

a. Prove that there are two vertices in A with exactly the same neighbors in B.

b. Prove that there are three vertices in A with exactly the same neighbors in B.