Combinatorics and Graph Theory II

Midterm 1 makeup, December 6, 2023, 12.15-13.45

You have 90 minutes for the problem set. Please write your **name** and **neptun code** on each sheet. Each problem is worth 10 points. Results without proper argument are not marked. Partial results get partial points. You need to get 24 points to pass.

Pen, paper and problem sets can be used, nothing else.

Good luck!

1. G is a plane graph (a graph drawn in the plane with no crossings) of 100 vertices. All faces are triangles, including the outer face, except one face, which is a pentagon. Determine the number of edges of G.

2. G is a graph with the property that if we remove ANY two of its edges, the resulting graph is planar. Show that $\chi(G) \leq 5$.

3. Suppose that G is a simple plane graph, isomorphic to its dual. Prove that G is not a bipartite graph.

4. Let $n > 0$, The vertices of G are $v_1, v_2, \ldots, v_n$. Vertices v_i and v_j, $i \neq j$, are connected by an edge if $|i - j|$ is divisible by 4. For which values of n will G be perfect?

5. Prove that $ch(K_{100,100}) > 3$.

6. Let $n > 0$, The vertices of G are $v_1, v_2, \ldots, v_n$. Vertices v_i and v_j are connected by an edge if $i > 2^j$ or $j > 2^i$. Is G perfect?

Combinatorics and Graph Theory II

Midterm 2 makeup, December 6, 2023, 12.15-13.45

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Pen, paper and problem sets can be used, nothing else.

Good luck!

1. Prove that for any $k \geq l \geq 2$ there is a graph G with $ch(G) = k$ and $\chi(G) = l$.

2. a. Prove that there is a number R with the following property. If we color the edges of K_R with three colors, there is always a triangle whose edges are colored with at most two colors.

b. Prove that the smallest such R is 5.

3. Let P_3 be the path of length 3 (three edges) and let $n \geq 3$. Determine the maximum number of edges of a graph of n vertices which does not contain a triangle and P_3 as a subgraph.

4. Prove that for any r there is an $N(r)$ with the following property. For any r-coloring of the numbers $1, 2, \ldots, N(r)$, there are four distinct numbers, a, b, c, d, of the same color such that $a + b = c + d$.

5. Suppose that $\mathcal{F} \subseteq 2^{[10]}$ is a simple set system (one subset can appear at most once), any two of the sets have a nonempty intersection and all sets have 4, 5, or 6 elements. Prove that $|\mathcal{F}| \leq \binom{9}{4} + \binom{10}{4}$.

6. Suppose that $\mathcal{F} \subseteq 2^{[n]}$ is a simple set system (one subset can appear at most once). We know that if $A, B \in \mathcal{F}$ and $A \subset B$, then $||A| - |B|| = 1$. Prove that

$$|\mathcal{F}| \leq 2\binom{n}{\lfloor n/2 \rfloor}.$$