Things to know:

$G$ is a graph and there is a list of colors $L(v)$ assigned to every vertex $v$ of $G$. $G$ is $L$ colorable if there is a proper coloring of $G$ such that for every vertex $v$ its color $c(v) \in L(v)$. The list coloring number of $G$, $\text{ch}(G)$ is the smallest $k$ with the property that whenever $|L(v)| = k$ for every $v$, then $G$ is $L$ colorable.

For every $G$, $\chi(G) \leq \text{ch}(G)$, and for every $k$ there is a graph $G$ with $\chi(G) = 2$ de $\text{ch}(G) > k$.

For every $G$, $\chi(G) \leq \Delta(G) + 1$.

List coloring conjecture: If $G$ is a line graph, then $\chi(G) = \text{ch}(G)$.

Thomassen 94: If $G$ is a planar graph, then $\text{ch}(G) \leq 5$.

Voigt 93: There is a $G$ planar graph for which $\text{ch}(G) = 5$.

1. Determine $\text{ch}(K_{2,4})$. ($\chi(K_{2,4})$ is a complete bipartite graph with 2 and 4 vertices in the classes.)

2. Is it true that if $\chi(G) = \text{ch}(G)$, then $\text{ch}(G) = 2$?

3. There is a list of colors on each vertex of $G$. Each list has length at least $\text{ch}(G)$. Is it true that for a proper ordering of the vertices it can be colored in a greedy way from the list? That is, we color in the given order and always choose the smallest possible color from the list.

4. Let $K_{2,2,...,2}$ a graph of $2n$ vertices whose complement is $n$ independent edges. Determine $\text{ch}(K_{2,2,...,2})$.

5. Show a graph which is not a line graph.

6. Show that if $G$ is a line graph, then $\text{ch}(G) \leq 2\chi(G) - 1$.

7. Suppose that for a graph $G$ and lists $L$, $|L(v)| > d(v)$ holds for every vertex $v$. Prove that $G$ is $L$-colorable.

8. Show that for any tree $T$ (of at least 2 vertices) $\text{ch}(T) = 2$.

9. Prove that if $G$ is a graph and $L$ is a list of colors on each vertex of $G$. Each list has length at least $\text{ch}(G)$. Is it true that for a proper ordering of the vertices it can be colored in a greedy way from the list? That is, we color in the given order and always choose the smallest possible color from the list?

10. For an arbitrary graph $G$, let $3G$ be the following graph. We take three disjoint copies of $G$ and connect the corresponding vertices in the three copies. Prove that if $G$ is planar, then $\text{ch}(3G) \leq 7$.

11. $G$ is drawn in the plane with just one edge crossing (where exactly two edges cross) Prove that
   a. $\text{ch}(G) \leq 6$,
   b. $\text{ch}(G) \leq 5$.

12. $G$ is a planar graph with at least 4 vertices. We have 7 colors, $\{1, 2, \ldots, 7\}$, we want to color $G$ with these colors. But somebody already colored 4 vertices that span a $K_4$ with colors 1, 2, 3, 4 Prove that we can finish the coloring, that is, we can color the rest of the vertices with colors $\{1, 2, \ldots, 7\}$ (so that neighboring vertices get different colors)

13. If we remove any 4 edges from $G$, we get a planar graph. Prove that $\text{ch}(G) \leq 6$.

14. $G$ is a planar graph. There is a list of five colors at each vertex, except one vertex where we have a list of one color. Prove that $G$ can be colored from the lists.

Homework

1. Prove that $\text{ch}(K_{n,n}) = n + 1$ for every $n > 0$.
2. Prove that if $C$ is an even cycle then $\text{ch}(C) = 2$. 

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