BSM, Real Functions and Measures, Fall 2014

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Web page of the course: www.renyi.hu/~harangi/rfm/

Class: Wednesday 14:15-16:00 (room 104), Thursday 14:15-16:00 (Room 206).
Office hours: last 20-25 minutes of each class.

Textbook: *Real and Complex Analysis, Third Edition* by Walter Rudin (1987, McGraw-Hill Book Company).

Homework: Homework problems are assigned every week. They are due the following Thursday.
You are allowed to collaborate with other students while working on these problems. However, I ask you not to tell each other complete solutions. Also, you must write up your solutions on your own.

Grades: For grading the exams I plan to use the following scale.
70%-100%=A ; 51%-69%=B ; 35%-50%=C ; 20%-34%=D.
In your final grade the homework, the midterm and the final exam count as 10%, 40% and 50%, respectively.

Prerequisite: calculus and an introductory analysis course; some elementary knowledge of topology and linear algebra is desirable, but a short introduction will be offered to make the course self contained. (Please consult the syllabus of the ANT course; if most of the material it covers is unfamiliar to you, then take that course instead.)

Course description: Abstract measure theory and integration on measure spaces was developed in the late 19th and early 20th century. This was the time when it became clear to many mathematicians that the Riemann integral (about which one learns in calculus courses) should be replaced by some other type of integral. So what's wrong with the Riemann integral? It's not general or flexible enough (only fairly nice functions are Riemann integrable), and it's not well suited to deal with limits.

The first step in the theory was the construction of the Lebesgue measure on the real line and the development of the corresponding Lebesgue integration. (As we will see it in this course, this new concept of integration keeps the advantages of the Riemann integral and eliminates its drawbacks.) Then it turned out that the idea of Lebesgue integration can be immensely generalized. This abstract theory is not in any way more difficult than the special case of the real line; it shows that a large part of integration theory is independent of any geometry (or topology) of the underlying space; and, of course, it gives us a tool of much wider applicability. This general concept of integration later became the basis of contemporary analysis, as well as of Kolmogorov's axiomatisation of probability theory, and ergodic theory. It has become a "language" that every mathematician needs to speak. This course provides an introduction to the subject with special emphasis on problem solving.

Topics: Topological and measurable spaces. The abstract theory of measurable sets and functions, integration. Borel measures, linear functionals, the Riesz theorem. Bounded variation and absolute continuity. The Lebesgue-Radon-Nikodym theorem. The maximal theorem. Differentiation of measures and functions. Density.