3D Convex hull Algorithm, cumulative time to refresh the conflict graph:

For every horizon edge we need to check the points that see one of the faces incident to this edge in $C(P_r)$ and connect it with the appropriate ones.

Definition: if e is an edge of $C(P_r)$, then let $K_r(e)$ = [set of points in $(P \setminus P_r)$ that see e]=[set of points in $(P \setminus P_r)$ that see at least one face incident to e].

So we need to bound the following (r is the index of a step in the algorithm):

$$E(\sum_{r=1}^{n} \sum_{[e \text{ on the horizon in step r+1.}]} |K_r(e)|)$$

We can upper bound this by summing instead $|K_r(e)|$ for every edge of $C(P_{r+1})$ and then take c/r times this. Indeed, above we sum only for horizon edges and an edge of $C(P_{r+1})$ becomes a horizon edge with probability $c/(r+1) < c/r$ in step $(r+1)$. by backwards analysis: for a fixed edge out of the $(r+1)$ points only two are such that if we choose them as p_{r+1} then e becomes an edge on the horizon).

A fixed point q is in $K_r(e)$ exactly for those e for which, when adding q to P_r and construction the convex hull of this point set, then e would end up on a face with q or would disappear. Thus:

$$\leq E(\sum_{r=1}^{n} \frac{c}{r} \sum_{q \notin P_r} [[\text{degree of } q \text{ in } C(P_r \cup \{q\})] + [\text{number of hull edges disappearing during } P_r \to P_r \cup \{q\}]])$$

Summing over q is the same as taking a random q and then multiplying by $(n-r)$. Notice that p_{i+1} plays the role of a random q. Moreover, the number of edges disappearing in step r is at most c times more than the number of disappearing faces (using Euler's formula on the planar graph of the disappearing edges plus horizon edges). Thus:

TODO

$$\leq \sum_{r=1}^{n} \frac{c}{r} c'(n-r)[E(\text{degree of } p_{r+1} \text{ in } C(P_{r+1})) + E(\text{number of hull faces disappearing during } P_r \to P_{r+1})]$$

Analysis of E(number of hull edges disappearing during $P_r \to P_{r+1}$)]: we can sum instead over every face, then for a fix Δ face its contribution is $c''(n-r)/r$, where r is the step when Δ disappeared. The step i when Δ appeared, has $i < r$, thus $(n-i)/i > (n-r)/r$, therefore changing inthe sum the contribution of this face to $c''(n-i)/i$, we get a larger sum. This is true in any fixed case, so it is true also for the expectation, so we can assign Δ to the step where it appeared and then going back to summing over r we get:

$$\leq \sum_{r=1}^{n} \frac{c}{r} c'(n-r)[E(\text{degree of } p_{r+1} \text{ in } C(P_{r+1}) + E(\text{number of hull faces } \textbf{appearing} \text{ during } P_r \to P_{r+1}]]$$

By backwards analysis the expected degree of p_{r+1} is at most 6 (the expected degree of a random vertex in a planar graph), the number of faces that appear in the $r+1$.th step is the same, and thus altogether:

$$\leq \sum_{r=1}^{n} c''(n-r)/r = O(n \log n).$$