1 Second exercise set

Fundamental

E 1. Show that on $\mathbb{Z}$ every bounded harmonic function with respect to the simple random walk is constant.

E 2. Construct a non-constant bounded harmonic function on the 3-regular tree T_3 (again, with respect to the simple random walk).

E 3. Show that if an irreducible Markov chain is recurrent, then it has the *Liouville property*, i.e. all bounded harmonic functions are constant.

On topic

E 4. Show that for any non-degenerate step-distribution μ on an Abelian group G the random walk is Liouville.

E 5. Show that the trajectory of the simple random walk on the d-regular tree T_d ($d \geq 3$) converges to a point on the boundary ∂T with probability 1.

Connected

E 6. Consider the simple random walk on $\mathbb{Z}$ between a and b, stopped at the endpoints. Condition the walk on ending up at b. Show that after the conditioning the random walk is still a Markov chain, and determine the transition probabilites.

E 7. Condition the simple random walk on T_d to converge to a given boundary point. Describe this new random walk directly!