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AN ADDENDUM TO OUR PAPER "FURTHER REMARKS ON δ- AND θ-MODIFICATIONS"

E. MAKAI, JR.*

Alfréd Rényi Mathematical Institute, H-1053 Budapest, Reáltanoda u. 13–15, Hungary
http://www.renyi.hu/~makai
e-mail: makai@renyi.hu

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Abstract. We show functoriality of γ^α in our joint paper with Á. Császár "Further remarks on δ- and θ-modifications".

Let X, Y and f be like at Proposition 2.11 of [1], let $A \subset X$, let α be an ordinal and let γ^α be like before Theorem 4.5 of [1].

PROPOSITION. *If $f : X \to Y$ is (μ, ν)-continuous and (μ', ν')-continuous, then $f(\gamma^\alpha A) \subset \gamma^\alpha\big(f(A)\big)$.*

PROOF. We begin with the case $\alpha = 1$. Let $x \in X$, $y := f(x)$ and $B := f(A)$. Let $y \notin \gamma(B)$, i.e., there exists $y \in N \in \nu$, such that $c_{\nu'}(N) \cap B = \emptyset$. Then $x \in f^{-1}(N) \in \mu$, and $f^{-1}\big(c_{\nu'}(N)\big) \cap f^{-1}(B) = \emptyset$. Here $f^{-1}\big(c_{\nu'}(N)\big)\ \big(\supset f^{-1}(N)\big)$ is μ'-closed, hence $c_{\mu'} f^{-1}(N) \subset f^{-1}\big(c_{\nu'}(N)\big)$, and $A \subset f^{-1}(B)$. Hence $c_{\mu'}\big(f^{-1}(N)\big) \cap A = \emptyset$, showing $x \notin \gamma A$. This proves $f(\gamma A) \subset \gamma f(A)$.

Now we use transfinite induction, with induction base $\alpha = 0$, for which the statement is evident. For $\alpha = \beta + 1$ a successor ordinal we have $f(\gamma^\beta A) \subset \gamma^\beta\big(f(A)\big)$, hence $f(\gamma^\alpha A) = f(\gamma\gamma^\beta A) \subset \gamma f(\gamma^\beta A) \subset \gamma\gamma^\beta f(A) = \gamma^\alpha f(A)$. For α a limit ordinal the induction step is evident. $\square$

References

[1] Á. Császár and E. Makai, Jr., Further remarks on δ- and θ-modifications, *Acta Math. Hungar.,* **123** (2009), 223–228.

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