# SOME PERSONAL AND MATHEMATICAL REMINISCENCES OF KURT MAHLER

**Paul Erdős**

I met Kurt Mahler first at the international mathematical congress in Oslo. I knew of his work many years earlier and was very glad to meet him. I almost immediately posed him the following problem: An integer is called squarefull or powerful if $p \mid m$ implies $p^2 \mid m$; are there infinitely many consecutive powerful numbers? Mahler immediately answered: Trivially, yes! $x^2 - 8y^2 = 1$ has infinitely many solutions. I was a bit crestfallen since I felt that I should have thought of this myself. But I then asked: Is it true that the number of pairs of consecutive powerful numbers not exceeding $x$ is equal to $c \log x$—and if this would not be true give as good as possible upper and lower bounds for the number of pairs of consecutive powerful numbers. As far as I know these problems are still open. Many related questions can be posed; for example, three consecutive numbers can never be all powerful.

In 1937–38, we were together in Manchester for more than a year. We wrote two joint papers, had many mathematical and political discussions, walked a great deal (despite his poor health, Mahler liked to walk very much) and we also played bridge.

First about mathematics. Let $f(x, y)$ be a binary form of degree $k$. Mahler and I proved that the number of distinct integers not exceeding $n$ which can be written in the form $f(x, y)$ ($x$ and $y$ are of course integers) is between $c_1 n^{2/k}$ and $c_2 n^{2/k}$. In particular, the number of integers of the form $x^k + y^k$ is greater than $cn^{2/k}$. This sharpened an older result of Landau. Later Hooley proved that the number of integers not exceeding $n$ of the form $f(x, y)$ is $((c + o(1))n^{2/k}$. One would of course expect that an analogous result holds for ternary forms, but even the very special case $x^3 + y^3 + z^3$ is intractable and I expect that it will stay so for a long time.

Our second joint paper dealt with continued fractions (*J. London Math. Soc.*, **14** (1939), 77–181). We proved that for almost all $\alpha$ there are only finitely many integers $n$ for which

$$P(B_n) < \exp\left(\frac{\log B_n}{\log\log B_n}\right)$$

where $A_n/B_n$ are the convergents of $\alpha$ and $P(m)$ is the greatest prime factor of $m$. We conjectured that if $P(A_n \cdot B_n) < c$ for infinitely many $n$ then $\alpha$ is a Liouville number.

This paper seemed dead, but recently some of our results were sharpened, I believe by Shorey.

Let me tell an amusing anecdote of our time in Manchester. Mahler was an enthusiastic but fairly poor bridge player, even by my very moderate standards. Davenport, Ko, Zilinskas and I played bridge fairly regularly, maybe less than one hour a day and sometimes Mahler joined in. Once my partner Ko did something which I thought was wrong. I told him, Ko you play $O(M)$, that is, order(Mahler). Davenport suggested that when Mahler comes we should tell him that $O(M)$ means obvious mistake. In fact it worked; Mahler did not guess. One evening there was a large bridge party at the Mordells. Ko suggested $O(D)$ = obviously down, $O(E)$ = obvious error, $O(K)$ = all correct and $O(H)$ = obviously harmful (Davenport, Erdős, Ko, Heilbronn). In September 1938 when I told him the awful truth he took it in very good humour.

We often corresponded while I was in the US on various mathematical problems mostly on diophantine equations and diophantine approximations. Mahler did his very important work on geometry of numbers in the late forties. I only admired this work from the distance and never worked seriously on it.

I often visited him in Manchester in the late 1940s and 50s. Mahler went to Canberra in 1962 but I met him there first around 1976. In 1970, my mother and I met him in Columbus, Ohio. Besides mathematics, Mahler was interested in linguistics and learned Chinese and even read simple texts. If I remember right there was a plan that he should go to China in the early 1940s, but shortage of shipping prevented this plan.

I visited Australia fairly often and of course always visited Mahler in Canberra. We had many mathematical discussions but he could no longer walk a great deal. I was in Australia for $2\frac{1}{2}$ months early in 1988 and in Canberra in February. I had dinner and lunch with Mahler at University House and met him in the Department of Mathematics of the Institute for Advanced Studies. He was clearly frail but I did not expect that the end was so near. He told me about his last paper. Let $\epsilon_i = 0$ or 1. Then

$$\sum_{i=1}^{n} \epsilon_i k^i = x^2$$

has infinitely many solutions for $k = 2$, 3 and 4. He conjectured that it has only a finite number of solutions for $k > 4$. In fact, the only nontrivial solution he found was $7^3 + 7^2 + 7 + 1 = 20^2$, and perhaps there are no others $(1 + (k^2 - 1) = k^2$ counts as trivial). I thought that perhaps

$$\sum_{i=1}^{n} \epsilon_i i! = x^2 \quad \text{or} \quad \sum_{i=1}^{k} \epsilon_i i! = x^k$$

has only a finite number of solutions. In full generality, this is hopeless since in the near (or distant) future nobody will know that $1 + n! = x^2$ has the only solutions $1 + 4! = 5^2$, $1 + 5! = 11^2$, $1 + 7! = 71^2$. Brindza and I proved some special cases and our paper will be dedicated to his memory.

After I returned to Sydney I learned with great sorrow that my old friend and co-worker Kurt is no more. May his theorems live forever!

Mathematical Institute
The Hungarian Academy of Sciences
Budapest, POB 127, H-1364

## DEATH OF ANDREW HURLEY

Dr A. C. Hurley, FAA, a member of the Australian Mathematical Society since 1957, died peacefully in his sleep on Friday 14 October 1988 after a long struggle with emphysema. He was born on 11 July 1926 and had only recently retired as Chief Research Scientist in the CSIRO Division of Chemical Physics where he had worked since 1953. Andrew achieved an international reputation for his theoretical contributions to the quantum theory of atoms and small molecules and was particularly interested in applied group theory. He will be sorely missed as a colleague and our profound sympathy goes out to his wife and family in their great loss. A more extensive account of Andrew's career will appear in *The Australian Physicist*.