Combinatorial Methods - list of theorems for the exam

1. Interval systems. Helly property. Transversal and matchings, their relation in arbitrary set systems. Algorithm to determine τ and ν for interval systems.
2. Interval systems. Decomposition into intersecting subsystems. Algorithm to find a decomposition into a minimum number of matchings.
3. Interval graphs. Induced cycles in interval graphs. First fit coloring for any graph.
4. Coloring number. $\chi(G) \leq \mathrm{col}(G)$. Algorithm determining the coloring number.
5. Helly propeerty of subtrees of a tree. Chordal graphs. Simplicial vertex, simplicial order. Equivalent definitions of chordal graphs.
6. Algorithms in chordal graphs. $\alpha, \theta, \chi, \omega$.
7. Tree-width. Tree decompositions. Tree decomposition with respect to chordal supergraph. Nice tree decomposition.
8. Algorithm to recognize if a graph is bipartite. Matchings in bipartite graphs. Alternating and augmenting paths. Transversals.
9. Algorithm to find a maximum matching a minimum transversal.
10. Hall's theorem. Application to distinct representatives of set systems.
11. Stable matchings, blocking edge. Algorithm to obtain a stable matching.
12. Maximum cut: there exists a cut containing half of the edges (2 algorithms, and a probabilistic proof).
13. Precoloring, list coloring. Relation to the coloring number. Example showing $\chi(G)$ and $\chi_\ell(G)$ can be different.
14. Kernel of directed graphs. Existence of a kernel for directed acyclic graphs and directed bipartite graphs. Application to list colorings of graphs.
15. Edge decompositions of complete graphs to matchings, Hamiltonian paths, Hamiltonian cycles.
16. Edge decompositions of complete graphs to complete graphs of fixed size (without proof), to complete bipartite graphs of variable sizes.
17. Projective planes of order q. Axioms and consequences.
18. Projective planes of order q: construction for prime power q.
19. Forbidden subgraphs. Mentel's theorem, Turán's theorem.
20. Forbidden subgraphs. C_4-free graphs.

Exam:

- Part I: one from the above list is randomly assigned. Proofs are required for grades 4 and 5.
- Part II: 6-8 definitions and statements (without proof) are asked from any of the above topics. Not knowing 2-3 of these definitions / statements results in failing the exam.

Consultation: online, 2 days before the exam, at request to patkosb at gmail dot com