Math Logic Homework-1.

1. (A) Let $\mathcal{A} = \langle \mathbb{R}, +, \cdot, 0, 1 \rangle$ be the usual ring of real numbers. Give a formula $\varphi(x, y)$ in the language of $\mathcal{A}$ so that for any evaluation k over $\mathcal{A}$ we have

$$\mathcal{A} \models \varphi[k] \quad \text{iff} \quad k(x) < k(y).$$

(B) The first order language L of partial orders contains only one binary relation symbol $<$. Give a first order formula ψ of L such that for any partial order $\mathcal{A} = \langle A, < \rangle$

$$\mathcal{A} \models \psi \quad \text{iff} \quad \text{any two elements of } A \text{ has a least upper bound.}$$

In this exercise you do not have to prove anything, but please be careful: you should provide well formed formulas (wff's) in the specified first order language.

2. Show that there is an evaluation k such that $k \models (X \Rightarrow Y) \Rightarrow Z$ but $k \not\models X \Rightarrow (Y \Rightarrow Z)$.

3. Let $\varphi = \exists x \exists y \exists z (P(x, y, z)) \wedge \forall x \forall y \forall z (P(x, y, z) \Rightarrow P(y, z, x))$. Give a first order structure $\mathcal{A}$ such that $\mathcal{A} \models \varphi$.

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