Exercises from Previous Tests

Combinatorics:
Transportation network problems
Higher order connectivity

Probability:
Law of Large Numbers,
Central Limit Theorem.

Combinatorics

0. Practice to find maximum valued flows in transportation networks. You may find concrete exercises of this kind (by Rita Csákány) at

http://www.cs.bme.hu/˜csakany/cgt1/gy2110.pdf

Concentrate to the simpler questions in "exercise 1".

1. Is there exists a 2016-connected graph which is 2000-regular? (Prove your statement, a simple yes/no answer is not enough).

2. Let V be the 3-element subsets of $\{1, 2, ..., 2016\}$ and let $\mathcal{G} = \langle V, E \rangle$ be the graph in which $(a, b) \in V^2$ forms an edge iff $a \cap b \neq \emptyset$.

(a) Suppose $a, b \in V$ are disjoint. How many common neighbors do a and b have?

(b) Prove that $\mathcal{G}$ is at least 18-connected.

3. Show that if $\mathcal{G} = \langle V, E \rangle$ is 6-connected, then $3|V| \leq |E|$ (that is, the number of edges of $\mathcal{G}$ is bigger than, or equal with, three times the number of vertices of $\mathcal{G}$).

4. Show that each 3-regular, 3-edge-connected graph is also 3-connected.

Probability

1. People are taking out money from a bank automata (ATM machine). The amounts they are taking out can be regarded as independent, random quantities with expected value 100000 (one-hundred-thousand) HUF and variance 20000 (twenty-thousand) HUF. At the beginning of the day the machine contains 6 million HUF. Estimate the probability that the 50^{th} people can take out 400000 HUF ((s)he stores more money in his/her bank account).

Solution. Let ξ_i be the amount the i^{th} people takes out. Then the first 49 people altogether take out $\sum_i^{49} \xi_i$ HUF, so we have to estimate

$$P(\sum_i^{49} \xi_i \leq 5600000).$$

To do so, apply the Central Limit Theorem.

2. Consider the ATM machine in the previous exercise. People are asking for a printed invoice about their transactions with probability $p = 0.2$ and with independently from each other. Estimate the probability that after serving the 1000^{th} people, the number of printed invoices will be between 170 and 230.

Solution. Apply the Law of Large Numbers.

3. Controllers in public transportation find a passenger without a valid ticket with probability $p = 0.02$.
(a) Let ν be the number of passengers without a valid ticket among the next 1000 passengers the controllers check. Estimate the probability, that ν is in between 16 and 24.
(b) Which is the probability, that the n^{th} checked passenger will be the 2^{nd}, who does not have a valid ticket?

Solution. For (a) apply the Law of Large Numbers. The answer for (b) is

$$p \cdot \binom{n-1}{1} p^1 (1-p)^{n-2}.$$

4. In a fast-food restaurant people order cheeseburger with (unknown) probability p. We want to estimate the value of p with the relative frequency of cheeseburger buyers among the next n customers. How to choose n if we would like to keep the difference between p and the relative frequency below 0.05 with probability at least 0.99?

Solution. Apply the Law of Large Numbers.

5. In the previous fast-food restaurant 25 customers are waiting in a long queue. The amount of time for serving customers are independent random variables with expected value 1 minute and variance 0.5 minute. Estimate the probability that the 25^{th} customer will be served during the next 30 minutes.

Solution. Apply the Central Limit Theorem.

7. In the previous fast-food restaurant customers may drink as many soft drinks as they want. The amount of soft drink needed by a customer is a random variable with expected value 3 dl and variance 1 dl. Estimate the probability that 32 liters soft drink will be enough for the next 100 customers.

Solution. Apply the Central Limit Theorem.

8. In a car service the amount of time when an expert verifies a car is a random variable with expected value 30 minutes and variance 10 minutes. The price for such a general checking is 5000 HUF. Estimate the probability that after 100 checkings, the hourly rate of the expert is at least 9375 HUF.

Solution. Apply the Central Limit Theorem.

Some values of the Φ function

$\Phi(0) = 0,5,\quad \Phi(1) = 0,8413\quad \Phi(2) = 0,9772\quad \Phi(3) = 0,9987,$
$\Phi(4) = 0,9999\quad \Phi(5) = 0,9999$