Kähler quantization of the (co)tangent bundle

This is a joint work with László lempert.

Geometric quantization often produces not one Hilbert space to represent the quantum states of a classical system but a whole family H_s of Hilbert spaces, and the question arises if the spaces H_s are canonically isomorphic. Witten and Hitchin's work suggest to view H_s as fibers of a Hilbert bundle H, introduce a connection on H, and use parallel transport to identify different fibers. Here we explore to what extent this can be done. We introduce the notion of smooth and analytic fields of Hilbert spaces, and prove that if an analytic field over a simply connected base is flat, then it corresponds to a Hermitian Hilbert bundle with a flat connection and path independent parallel transport. We also address a general direct image problem in complex geometry: pushing forward a Hermitian holomorphic vector bundle $E \to Y$ along a non–proper map $Y \to S$. We give criteria for the direct image to be a smooth field of Hilbert spaces. Finally we consider quantizing an analytic Riemannian manifold M by endowing T^*M with the family of adapted Kähler structures. This leads to a direct image problem. When M is homogeneous, we prove the direct image is an analytic field of Hilbert spaces. For certain such M—but not all—the direct image is even flat; which means that in those cases quantization is unique.