**Homework 2**
due November 18, 2022.

1. Prove that $\chi_f(G) + \chi_f(\overline{G}) \geq 2\sqrt{n}$, where $n = |V(G)|$.

   [3 points]

2. Give a nowhere vanishing probability distribution $P = (p_1, \ldots, p_9)$ on the vertices of the 9-cycle $C_9$ for which

   $$H(C_9, P) + H(\overline{C_9}, P) = H(P).$$

   (The condition "nowhere vanishing" means that $p_i \neq 0$ for any $i \in \{1, \ldots, 9\}$.)

   [4 points]

3. Let $G$ be a graph with vertices $v_1, \ldots, v_n$ where the degree of vertex $v_i$ is $d_i$. Let $P$ be a probability distribution on the vertex set, and denote $P(v_i)$ by $p_i$. Prove that

   $$H(G, P) \leq \sum_{i=1}^{n} p_i \log(d_i + 1).$$

   [5 points]