

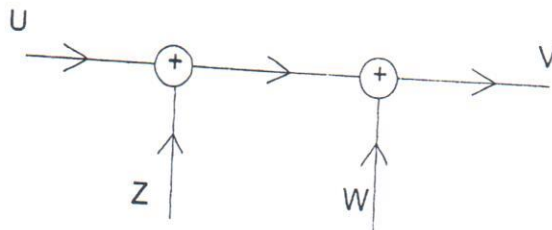
Information Theory
Third Midterm
November 23, 2017

- 1) Give the definition of quadratic distortion of a quantizer Q .
2. State the converse part of the Channel Coding Theorem
- 3) Let the random variable X have density function $f(x)$ given as follows.

$$f(x) = x + 1 \text{ if } x \in [-1, 0], \quad f(x) = -x + 1 \text{ if } x \in [0, 1],$$

and $f(x)$ is 0 outside the interval $[-1, 1]$. Perform the first iteration of the Lloyd-Max algorithm for a two-level quantizer of the variable X starting with initial quantization values $-0.5, 0.5$.

- 4) U, Z, W are independent binary (0-1 valued) random variables on the figure and \oplus means modulo 2 sum. The scheme shown by the figure gives a channel with binary input U and binary output V . What is the capacity of this channel if $P(Z = 1) = p$ and $P(W = 1) = q$?



- 5) Let the input alphabet of a discrete memoryless channel be $\mathcal{X} = \{0, 1, 3, 4\}$ and the output alphabet be $\mathcal{Y} = \{0, 1, 2, 3, 4, 5\}$. The transition probabilities are given by $W(i|i) = 1 - p$ for all $i \in \{0, 1, 3, 4\}$, $W(2|0) = W(2|1) = p$ and $W(5|3) = W(5|4) = p$. All other transition probabilities are equal to 0. (See figure below.) Give the capacity of this channel.

