Summary of what we covered and info for the Final.

In this course our major theme was learning some concepts/tools/theory for distinguishing *topological spaces* up to *homeomorphism*.

Thus you need to know the definitions of "a topological space", "homeomorphism of topological spaces" (and what open sets and continuous functions are), as well as examples of topological spaces.

In particular, (but not restricting to) special topological spaces such as the discrete/anti-discrete, arrow, co-finite, co-countable topologies. Various descriptions of the following surfaces (what is a surface?): sphere, cylinder, torus, Klein bottle. Also, the Mobius strip, which a surface with boundary. And "1-dimensional examples" such as intervals, $\mathbb{R}$, circle etc

You also need to know how new topological spaces are obtained from old by considering subspace, product, quotient constructions ("gluing"), as well as the connected sum of surfaces.

As for distinguishing topological spaces:

a.) to show that two topological spaces are homeomorphic (ie "the same", topologically) you need to provide an appropriate function (a homeomorphism) between them.

Note that continuous bijections are not necessarily homeomoprhisms. However, if you have a continuous bijection from a compact space to a Hausdorff space then it must be a homeomorphism.

In case of identification spaces (gluings) of polygons a cut-and-paste argument may be used to show homeomorphism.

Compact, connected surfaces having the same Euler characteristic and orientation are homeomorphic.

b.) We have more methods to show two spaces are *not* homeomorphic, since we studied several topological invariants: compactness, connectedness, path-connectedness, the Hausdorff property (coming from point-set topology), the fundamental group (coming from algebraic topology).

So you need to know definitions (and basic properties) of the above invariants, how they are related (e.g. path-connected implies connected, but not the other way around), which of these properties our example spaces have.

Of the above invariants we spent a considerable time studying the fundamental group. In particular, we have some results that help calculate it
1.) The "Very Important Corollary" provides a tool in distinguishing when two loops are homotopic, which in turn may be used to determine the fundamental group. It says that two paths that begin and end at the same point in some topological space X are *not* homotopic if there is a covering Y of X (and a covering map $p : Y \to X$) where one can find two lifts, one for each of the loops, that begin at the same point but end at different points.

Recall that given a covering $p : Y \to X$, the lift of a path $\alpha : [0,1] \to X$ is a path $\widetilde{\alpha} : [0,1] \to Y$ such that $\alpha = p \circ \widetilde{\alpha}$.

2.) As a consequence of 1.), if a topological space X has a simply connected cover Y ie $\exists p : Y \longrightarrow X$ covering with X, Y having basepoints $x_0 \in X$, $y_0 \in p^{-1}(y_0)$, then the mapping

$$\Phi : \pi(X, x_0) \longrightarrow p^{-1}(x_0)$$

where

$$\Phi(< \alpha >) = \widetilde{\alpha}(1)$$

is a bijection, if $\widetilde{\alpha}$ is the unique lift of α starting at y_0,

We use this fact to find the fundamental group of the circle, the projective plane, the torus and the figure eight.
3.) If A is a deformation retract of X, then they have the same fundamental group.
4.) We also have a formula $\pi_1(X \times Y, (x_0, y_0)) = \pi_1(X, x_0) \times \pi_1(Y, y_0)$.

The main result of the course is the classification of compact, connected surfaces. For these spaces we also studied a special pair of invariants, the orientability and Euler characteristic (equivalently "genus") - to help distinguish surfaces.

You only need to know/understand the statement of this theorem for the final and the "normal form" of how such a surface is represented as the identification space of a $2n$-gon. In addition, you have to know how to calculate and use the Euler characteristic and orientability to tell for a "random" compact, connected surface what it is homeomorphic to in the classification list; or distinguish such surfaces. (Alternatively, a cut-and-paste argument may be used for this.)

Please, look at applications as well, such as (examples of) obstructions on the existence of certain maps (homeomorphisms, retracts) or the Brouwer fixed point theorem.

The final will be comprehensive with an emphasis on the second half of the course.

It will be closed books and notes, but an A4 or "letter" size "cheat sheet" (carefully prepared before the final) can be used. You can write on both sides of it. The final will be on Friday, May 17th 8am-10am in 007.