We want to show that the projection maps

$proj_X : X \times Y \to X, \ (x,y) \mapsto x$

$proj_Y : X \times Y \to Y, \ (x,y) \mapsto y$

are open maps, that is: they take open sets to open sets, if $X \times Y$ has the product topology.

<u>Pf:</u> Consider $f = proj_X$ first.

Assume $W \subset X \times Y$ is an open set.

We want to show that

$Z = proj_X(W) \in \mathcal{T}_X$.

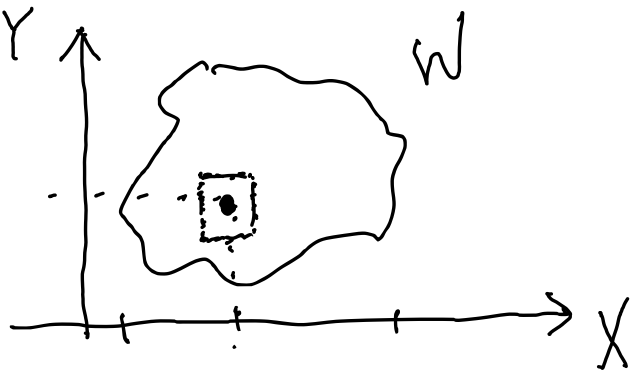
We will prove this by writing Z as a union of open sets.

Pick $a \in Z$. Then, since $a \in proj_X(W) = Z$

$\exists\, b \in Y$ s.t. $(a,b) \in W$.

Then, by definition of product topologies $\exists\, U_a \in \tau_X,\ V_a \in \tau_Y$ s.t.

$(a,b) \in U_a \times V_a \subset W$.



Clearly, $U_a = \mathrm{proj}_X(U_a \times V_a) \subset Z = \mathrm{proj}_X(W)$

Going over $a \in Z$, we get

$$Z = \bigcup_{a \in Z} U_a \in \tau_X,$$ since $U_a \in \tau_X$, $\forall a \in Z$