Clearly, this problem is only interesting if X has at least 2 points.
So we can pick $a, b \in X$ s.t. $a \neq b$.
Let $r = d(a,b)$, the distance between a and b. Then $r > 0$.
Now, consider the sets

$$A_s = \{ q \in X \mid d(a,q) = s \}$$

– the set of points in X that is distance s from a, for $\forall s \in (0, r)$.
Since there are uncountable many such s values and only countable many points in X, $\exists s_0 \in (0,r)$ s.t.

$A_{s_0} = \phi$ (in fact, uncountable many such s_0).

Then let

$$U = \{ q \in X \mid d(a,q) < s_0 \} = B_a(s_0)$$

$$V = \{ q \in X \mid d(a,q) > s_0 \}$$

Clearly, $a \in U$, $b \in V$ so $U, V \neq \phi$.

U is an open ball, so as we proved $U \in \tau_d$. Also $V \in \tau_d$ by a previous homework.

Clearly, $U \cap V = \emptyset$ and $X = U \cup V$.

So X is disconnected.