(1) Knowing if a compact, connected surface is orientable and its Euler characteristic, together determine the surface up to homeomorphism.

(a) We have the formula

$$\chi(M \# N) = \chi(M) + \chi(N) - 2$$

for M, N compact connected surfaces.

So $\chi(K \# T) = \chi(K) + \chi(T) - 2$
$= 0 + 0 - 2.$

Since K is nonorientable, contains a Möbius strip, so does $K \# T$.

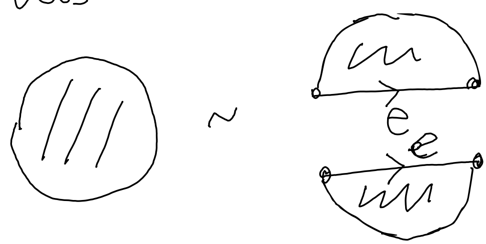
$\Rightarrow K \# T \sim \#_m \mathbb{RP}^2$

Since $\chi(\#_m \mathbb{RP}^2) = 2 - m$

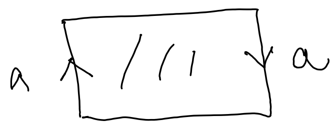
we have $2 - m = -2 \Rightarrow m = 4$

$\Rightarrow K \# T \sim \#_4 \mathbb{RP}^2$

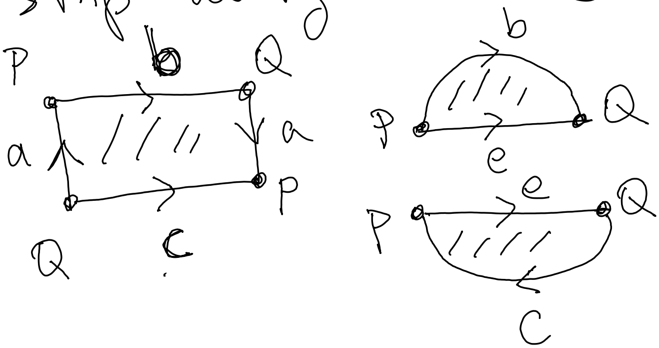
b.) Consider the disk

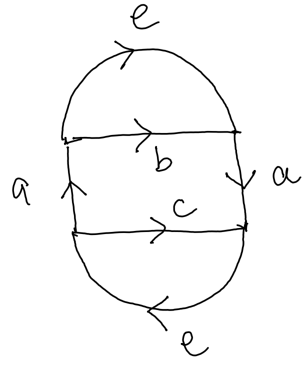
D^2 as 

The Möbius strip

is 

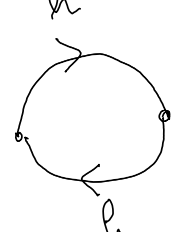
So the disk sewn onto the Möbius strip along the edge is



$\Rightarrow$ 

let

$h = ae$

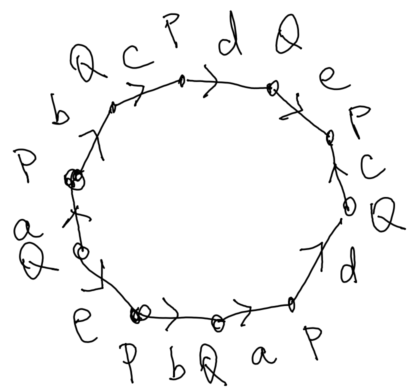
$\Rightarrow$ 

so we get

$\mathbb{RP}^2$

c.) Given $abcdec^{-1}d^{-1}a^{-1}b^{-1}e^{-1} = w$ it determines an orientable surface M since every edge is of first type.

The diagram corresponding to w determines a cell decomposition of the surface with



$\varphi^0 = \{P, Q\}$

$\varphi^1 = \{a, b, c, d, e\}$

$\varphi^2 = \{\text{the surface}\}$

$\Longrightarrow$

$\chi(M) = 2 - 5 + 1 = -2$

Since M is orientable

$M \sim \#nT$

Also $\chi(\#nT) = 2 - 2n \Rightarrow 2 - 2n = -2$

$n = 2$

$\Rightarrow M \sim \#2T = T \# T$