② a) Since there are V vertices and p edges meet at a vertex, pV gives the # of edges, however, calculated twice, since every edges is ends in two different vertices.

Thus $pV = 2E$. (1)

Similarly, since every face is an n-gon nF gives the number of edges, however each edge is calculated twice since it is part of two different faces.

So $nF = 2E$. (2)

b) Every platonic solid is homeomorphic to S^2, so their Euler number is 2

$\Rightarrow 2 = V - E + F$

Substituting $V = \frac{2E}{p}$ from (1) and $F = 2E/n$ from 2, we get

$$2 = \frac{2E}{p} - E + \frac{2E}{n}$$

$$\Rightarrow \frac{1}{E} = \frac{1}{p} - \frac{1}{2} + \frac{1}{n} \quad \ldots \quad (*)$$

In particular, we get $\frac{1}{p} + \frac{1}{n} > \frac{1}{2}$. (3)

c) We know $p, n \geq 3$. $\Rightarrow \frac{1}{p}, \frac{1}{n} \leq \frac{1}{3}$

Also $\frac{1}{p} + \frac{1}{n} > \frac{1}{2}$ implies

both $p, n < 6$. If eg. $p \geq 6$,

then $\frac{1}{p} \leq \frac{1}{6}$. On the other hand,

$\frac{1}{n} \leq \frac{1}{3}$ so $\frac{1}{p} + \frac{1}{n} \leq \frac{1}{6} + \frac{1}{3} = \frac{1}{2}$ ↯.

Similarly, $n \geq 6$ leads to a contradiction.

$\Rightarrow$ We have $3 \leq p, n < 6$

so all the <u>possible</u> values

are $p, n = 3, 4, 5$.

	P	n	
	3	3	tetrahedron
	3	4	cube
	3	5	icosahedron
	4	3	octahedron
a	4	4	no such
b	4	5	no such
	5	3	dodecahedron
c	5	4	no such
d	5	5	no such

Eg: "no such" solids are not possible since we know

$$\frac{1}{E} = \frac{1}{P} - \frac{1}{2} + \frac{1}{n}$$

and for a – d this equation does not work out for E to be an integer.