Consider, for example

$f : [0,1] \longrightarrow S^1$

$x \longmapsto (\cos 2\pi x, \sin 2\pi x)$

which is continuous & <u>onto</u>

However, the induced homomorphism

$f_* : \pi_1([0,1]) \longrightarrow \pi_1(S^1)$

cannot be onto, since

$\pi_1([0,1]) = 0$ & $\pi_1(S^1) \simeq \mathbb{Z}$.

<u>Also</u> consider the inclusion map

$i : S^1 \longrightarrow \mathbb{R}^2$ which is continuous and 1-1.

The induced homomorphism,

$i_* : \pi_1(S^1) \longrightarrow \pi_1(\mathbb{R}^2)$

cannot be 1-1, since $\pi_1(S^1) \simeq \mathbb{Z}$ and $\pi_1(\mathbb{R}^2) = 0$.