Let $n=3$ and choose any path

$\gamma : [0,1] \to \mathbb{R}$ s.t. $\gamma(0)=0$, $\gamma(1)=3$

e.g. $\gamma(s) = s(s+3)$

then

$p \circ \gamma(s) = (\cos 2\pi s(s+3), \sin 2\pi s(s+3))$

is a loop in S^1 at x_0

and $\gamma \simeq \tilde{\alpha}_3$, since in $\mathbb{R}$

the straight line homotopy

$H(s,t) = (1-t)\gamma(s) + t\, 3s$ will deform

γ to the lift of $\alpha_3(s) = (\cos 2\pi 3s, \sin 2\pi 3s)$

and so $p \circ H$ will deform

γ to α_3.