5.) Let $p: Y \to X$ be a covering and X connected.

Pick $b \in X$ and let

$$A_\lambda = \{ x \in X \mid |p^{-1}(x)| = |p^{-1}(b)| = \lambda \}.$$

Pick any $x \in A_\lambda$. Then, since p is a covering map, $\exists U_x \in \tau_X$, $x \in U_x$ s.t. U_x is evenly covered by sheets. In particular, p restricted to a sheet is a homeomorphism, so a bijection. So for all $y \in U_x$, each sheet above U_x contains exactly one pre-image of y.

Also, since $y \in U_x$, $p^{-1}(y) \subset p^{-1}(U_x)$, so there are no pre-images of y outside the sheets above U_x.

So $|p^{-1}(y)| =$ the cardinality of the set of sheets

$= |p^{-1}(x)|$.

So $y \in A_\lambda \Rightarrow U_x \subset A_\lambda \Rightarrow A_\lambda \in \tau_X$.

Now,

$$X = \dot{\bigcup_{\mu}} A_{\mu}\,, \quad \text{where } \lambda \text{ runs over all possible cardinalities.}$$

Since this gives X as the disjoint union of open sets and X is connected we must have $A_\lambda = X$ and all other sets empty. (Otherwise X would be disconnected.)