

The *projective plane* is a surface that is locally like a sphere, but has different global topology. It's made by gluing together the opposite points on the rim of a hemisphere (Figure 4.13). Figure 4.14 shows what this gluing looks like locally, along a short section of the rim. We can show the gluing along any section of the rim we like, but we can't show the entire gluing at once because of its peculiar global properties. Thus you should concentrate on understanding how opposite sections of rim fit together, rather than trying to visualize the whole thing at once the way you'd visualize

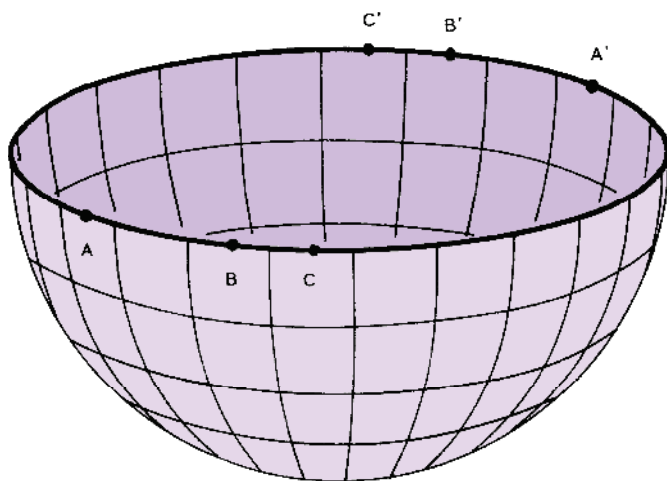


Figure 4.13: The projective plane is made by gluing together opposite points on the rim of a hemisphere.

a sphere. The most important thing is that the hemisphere's geometry matches up perfectly when opposite sections of rim are glued, so the projective plane has the same local geometry as a sphere, even along the "seams" where the gluing took place. The projective plane is our fourth homogeneous surface.

Exercise Is the projective plane orientable? That is, if a Flatlander crosses the "rim", does he come back normal or mirror-reversed?

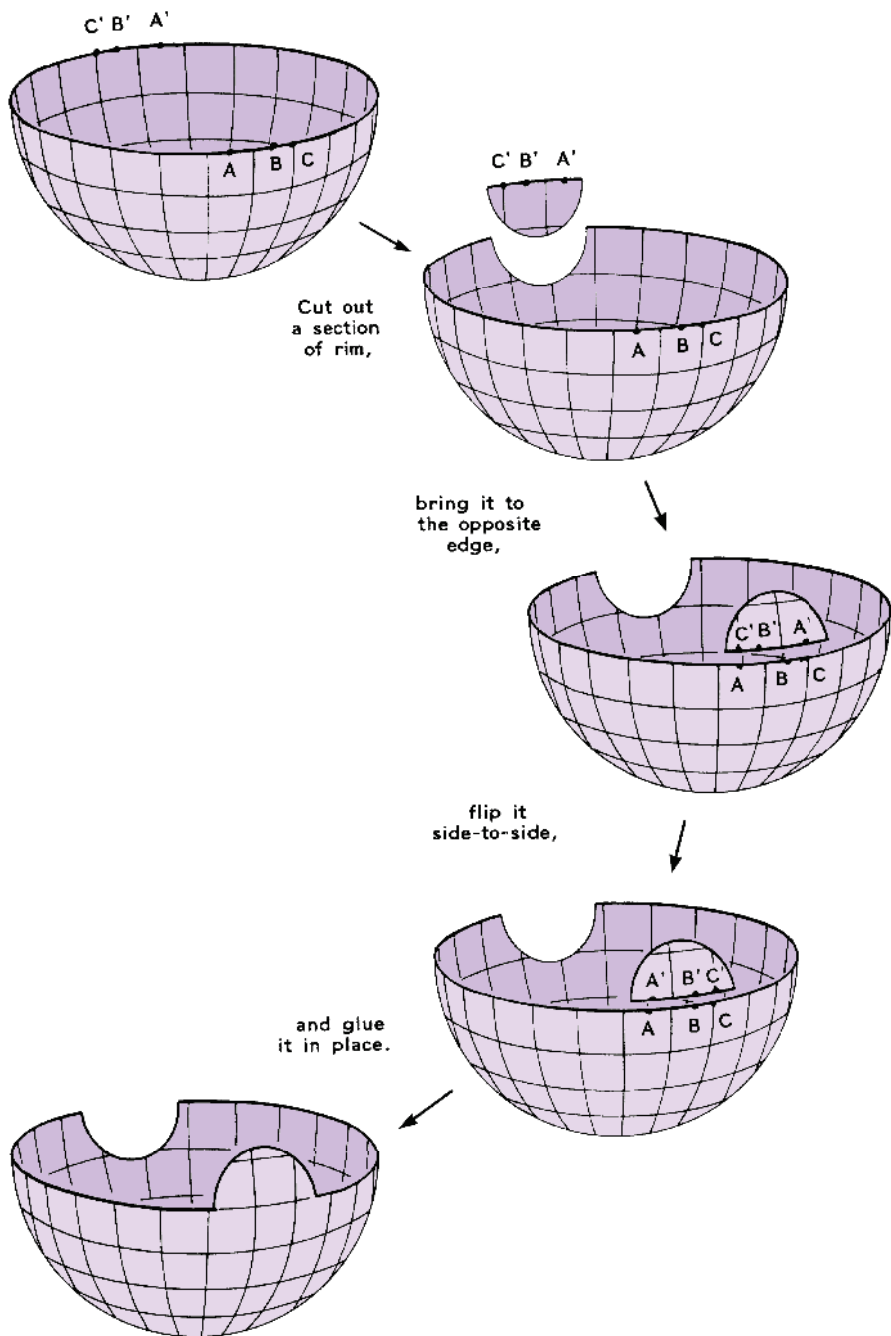


Figure 4.14: How to glue opposite sections of rim.

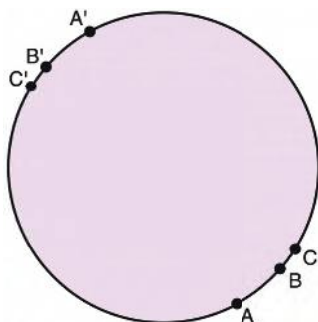


Figure 4.15: Topologically, a projective plane is a disk with opposite boundary points glued.

If we are interested in only the topological properties of the projective plane, we can flatten the hemisphere into a disk, still remembering to glue opposite boundary points (Figure 4.15). The main advantage of doing this is that a disk is easier to draw than a hemisphere.