#2 We have to show that if

U is closed in X

V is closed in Y

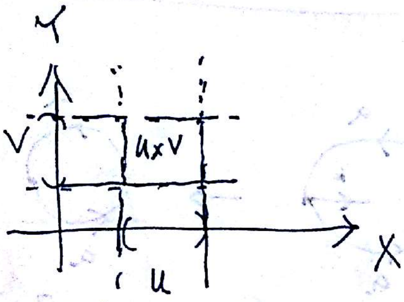
then $U \times V$ is closed in $X \times Y$,

<u>that is</u> if $X\setminus U \in \tau_X$, $Y \setminus V \in \tau_Y$

then $(X \times Y)\setminus(U\times V) \in \tau_{X\times Y}$.

<u>By properties of sets</u>

$$(X\times Y)\setminus(U\times V) = \left[(X\setminus U)\times Y\right] \cup \left[X \times (Y\setminus V)\right] \quad (*)$$



<u>Pf</u> $(a,b) \in (X\times Y)\setminus(U\times V)$

$\iff (a,b) \in X\times Y$ and $(a,b) \notin U\times V$

$\iff a\in X$ and $b \in Y$ and $a \notin U$ OR $b \notin V$

$\iff a \in X\setminus U$ OR $b \in Y\setminus V$ but still $a\in X$, $b\in Y$

$\iff (a,b) \in (X\setminus U)\times Y$ OR $(a,b)\in X\times(Y\setminus V)$

$\iff (a,b) \in ((X\setminus U)\times Y) \cup (X\times(Y\setminus V))$

However

$X\setminus U \times Y \in \tau_{X\times Y}$, since $X\setminus U\times Y \in \mathcal{B}$ already (~~since~~ see def of the product topology)

and similarly $X\times(Y\setminus V) \in \tau_{X\times Y}$, since it is in $\mathcal{B}$ already

$\Rightarrow$ RHS of (*) is in $\tau_{X\times Y}$, so the LHS as well.