① Show $cl(A \cup B) = cl A \cup cl B$

<u>method 1</u>: We show LHS $\supset$ RHS first.

Suppose $x \in (cl A) \cup (cl B)$. Then wlog assume $x \in cl A$

Then $\forall U \in \tau_x$, $x \in U$ we have $U \cap A \neq \phi$.

This implies $U \cap (A \cup B) \neq \phi$ either.

So $x \in cl(A \cup B)$.

Now show LHS $\subset$ RHS.

Assume not. That means $\exists x \in cl(A \cup B)$

s.t. $x \notin cl A$ <u>and</u> $x \notin cl B$.

If $x \notin cl A$, then $\exists V$ closed set in X s.t.

$A \subset V$ but $x \notin V$.

This is because

$cl A = \bigcap \{V \mid V \supset A,\ V \text{ closed in } X\}$

Similarly, if $x \notin cl B$, then $\exists W$ closed set in X

s.t. $W \supset B$, but $x \notin W$.

Consider $Z = V \cup W$. Then Z is a closed set,

since it is the finite union of closed sets.

Also, $A \cup B \subset Z$, clearly.

But $x \notin Z$, since $x \notin V$ <u>and</u> $x \notin W$.

So $x \notin cl(A \cup B)$ ↯.

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