Method 2

Every set is contained in its closure so we have $A \subset \text{cl} A$

and $B \subset \text{cl} B$.

Then by properties of unions

$$A \cup B \subset \text{cl} A \cup \text{cl} B.$$

Note that $F = \text{cl} A \cup \text{cl} B$ is a closed set, since it is the finite union of closed sets.

So $A \cup B \subset F = \text{closed}$, and by definition of closure we have $\text{cl}(A \cup B) \subset F = \text{cl} A \cup \text{cl} B$. (1)

Now, clearly $A \subset A \cup B$ and $B \subset A \cup B$

By monotonicity of closure (as shown in class) we have

$$\text{cl} A \subset \text{cl}(A \cup B) \quad \text{and} \quad \text{cl} B \subset \text{cl}(A \cup B)$$

Then by properties of unions we have

$$\text{cl} A \cup \text{cl} B \subset \text{cl}(A \cup B). \quad (2)$$

So, in fact, (1) & (2) imply

$$(\text{cl} A) \cup (\text{cl} B) = \text{cl}(A \cup B)$$