

#3. a) $cl A = [0, 3] \cup \{4\}$

$int A = (0, 1) \cup (1, 2)$

$\partial A = \{0, 1\} \cup [2, 3] \cup \{4\}$

b.) Both sides work out to be $[0, 2]$

c.) On one hand,

$int A \subset cl(int A)$, since $B \subset cl B \forall B$

Then, since $int A$ is an open set in $cl(int A)$, it has to be contained in its interior.

That is:

$int A \subset int(cl(int A))$.

But then $cl(int A) \subset cl(int(cl(int A)))$

This is because

$C \subset D \Rightarrow cl C \subset cl D, \forall C, D \text{ sets.}$

Argument: ~~$D \subset cl(D) \forall D.$~~

If also $C \subset D$, then $C \subset cl(D)$.

But $cl(D)$ is a closed set containing C , so by def of "closure", we must have

$cl C \subset cl D.$

On the other hand, $B \supset int B \forall B \text{ sets, so}$

$cl(int A) \supset int(cl(int A)).$

So we have $D = int(cl(int A))$ contained in a closed set, so $cl(D)$ must be in that set. So

$cl(int A) \supset cl(int(cl(int A)))$