2

(*) We know that in a (X, τ_X) Hausdorff space, if K is compact and $x \notin K$, then $\exists\, U, V \in \tau_X$, disjoint s.t. $x \in U$, $K \subset V$.

Now, we want to show that
if K, M are compact sets that are disjoint
then $\exists\, U, V \in \tau_X$, disjoint s.t. $K \subset U$, $M \subset V$.

So consider $z \in K$ and M.
By (*) we know $\exists\, U_z, V_z \in \tau_X$, disjoint
s.t. $z \in U_z$, $M \subset V_z$. Consider such U_z $\forall z \in K$.
Clearly, $\{U_z\}_{z \in K}$ is an open cover of K.
Since K is compact, $\exists$ a finite subcover
$\{U_1, \dots, U_n\} \subset \{U_k\}_{k \in K}$ i.e. $\bigcup_{i=1}^{n} U_i \supset K$.

Consider the corresponding $\{V_1, \dots, V_n\}$.
We know $M \subset V_i$ $\forall i \in \{1, \dots, n\}$.
So $M \subset V = \bigcap_{i=1}^{n} V_i$. Note that $V \in \tau_X$,
since V is the intersection of finite many
open sets.
Also, $U = \bigcup_{i=1}^{n} U_i$ is open, since it is the union
of open sets. We had $K \subset U$.
All there is remaining to show $U \cap V = \phi$.
Let $z \in U \cap V$. Then $z \in V_i$ $\forall i = \{1, \dots, n\}$.
Also $\exists j$ s.t. $z \in U_j$, since $U = \bigcup_{i=1}^{n} U_i$
But then $z \in U_j \cap V_j$ $\lightning$.