Practice on Homotopy. 1.

"F is a homotopy for $\alpha \simeq \alpha'$"

means:

we have $\alpha, \alpha' : [0,1] \to X$ continuous maps i.e. paths s.t.

$\alpha(0) = \alpha'(0) = a$ (starting point)

$\alpha(1) = \alpha'(1) = b$ (endpoint)

and

$F : [0,1] \times [0,1] \to X$ continuous is

s.t.

$$F(s,0) = \alpha(s) \quad \forall s \in [0,1]$$

$$F(s,1) = \alpha'(s) \quad \forall s \in [0,1]$$

$$F(0,t) = a \quad \forall t \in [0,1]$$

$$F(1,t) = b \quad \forall t \in [0,1]$$

For "G is a homotopy for $\beta \simeq \beta'$" is similar. Set $G(0,t) = b$,

$G(1,t) = c$.

b.) Given

$$H(s,t) = \begin{cases} F(2s,t) & \text{if } s \in [0,1/2] \\ G(2s-1,t) & \text{if } s \in [1/2,1] \end{cases}$$

we have

$$H(0,t) = F(0,t) = a$$

$$H(1,t) = G(1,t) = c$$

$$H(s,0) = \begin{cases} F(2s,0) & \text{if } s \in [0,1/2] \\ G(2s-1,0) & \text{if } s \in [1/2,1] \end{cases}$$

$$= \begin{cases} \alpha(2s) & \text{if } s \in [0,1/2] \\ \beta(2s-1) & \text{if } s \in [1/2,1] \end{cases}$$

$$= (\alpha * \beta)(s)$$

$$H(s,1) = \begin{cases} F(2s,1) & \text{if } s \in [0,1/2] \\ G(2s-1,0) & \text{if } s \in [1/2,1] \end{cases}$$

$$= \begin{cases} \alpha'(2s) & \text{if } s \in [0,1/2] \\ \beta'(2s-1) & \text{if } s \in [1/2,1] \end{cases}$$

$$= (\alpha' * \beta')(s)$$