Let $\varphi : S^2 \setminus \{N\} \to$ xy-plane

be the stereographic projection from

~~the~~ the North pole $N = (0,0,1)$.

Note that $\varphi \circ \alpha = \alpha$.

In the xy-plane the straight-line

homotopy

$$H : [0,1] \times [0,1] \longrightarrow \mathbb{R}^2 = xy\text{-plane}$$

where

$$H(s,t) = (1-t)\alpha(s) + t(1,0,0)$$

deforms α to $x_0 = (1,0,0)$.

Then $\varphi^{-1} \circ H : [0,1] \times [0,1] \to S^2 \setminus \{N\} \subset S^2$

deforms α to $x_0 = (1,0,0)$ in S^2.