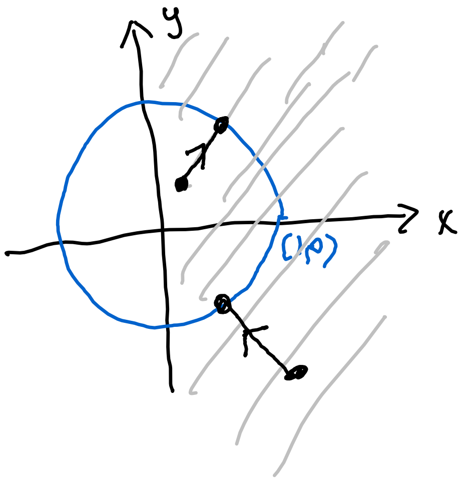
4.a. $X = S^1 \cup \{(x,y) \mid x > 0\} \subset \mathbb{R}^2$



can be deformation retracted onto $A = S^1$ via a radial straight line deformation in terms of Cartesian coordinates as:

$$H((x,y),t) = (1-t)(x,y) + t\frac{(x,y)}{\|(x,y)\|}$$

$$= \left((1-t)x + \frac{t}{\sqrt{x^2+y^2}}x,\ (1-t)y + \frac{t}{\sqrt{x^2+y^2}}y \right)$$

Since the coordinate functions are continuous, H is continuous. Also, by plugging in,

$H((x,y),0) = (x,y)$

$H((x,y),1) = \left(\frac{x}{\sqrt{x^2+y^2}}, \frac{y}{\sqrt{x^2+y^2}}\right) \in S^1 \quad \forall (x,y) \in X$

and if $(x,y) \in S^1$, then $\sqrt{x^2+y^2} = 1$, so

$H((x,y),t) = (x,y)$ so H is fixed on S^1.

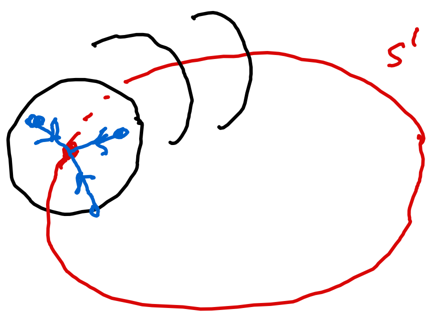
4b: $X = \{(x,y,u,v) \mid x^2+y^2 \le 1,\ u^2+v^2=1\}$

$$= \bar{D}^2 \times S^1 \subset \mathbb{R}^4$$

so for each $(u_0, v_0) \in S^1$ fixed

we have a closed disk centered at $(0,0,u_0,v_0)$

$\bar{D}^2 = \{(x,y,u_0,v_0) \mid x^2+y^2 \le 1\}$



Let

$A = \{(0,0,u,v) \mid u^2+v^2=1\}$

$\subset X$.

Clearly, $A \sim S^1$.

Let $G : X \times [0,1] \longrightarrow X$ be defined by

$$((x,y,u,v),t) \longmapsto ((1-t)x, (1-t)y, u, v)$$

Then G is continuous, since the coordinate maps are continuous, for $\underline{x} = (x,y,u,v)$

when $t=0$, $G(\underline{x}, 0) = (x,y,u,v) = \underline{x}$

when $t=1$, $G(\underline{x}, 1) = (0,0,u,v) \in A$

If $\underline{x} \in A$ i.e. $\underline{x} = (0,0,u,v)$ with $u^2+v^2=1$,

then $G(\underline{x}, t) = (0,0,u,v)$ so G fixes A pointwise.