

Consider $X = \{a, b, c\}$

$\tau = \{\emptyset, X\}$ and assume $\exists d$ metric on X that generates τ .

Fix $a \in X$ and consider the distances $r = d(a, b)$, $s = d(a, c)$.

Wlog, $r \leq s$.

Then let

$U = B_a(r/2)$ be the open ball centered at a , radius $r/2$.

Since $r/2 < r \leq s$, we have $c \notin U$,

and also $b \notin U$.

So $U = B_a(r/2) = \{a\}$.

However, we proved in class that every open ball is an open set.

So $U \in \tau$ must be true. But $U \neq \emptyset$ or X .

So d may not generate τ , but must have more sets.