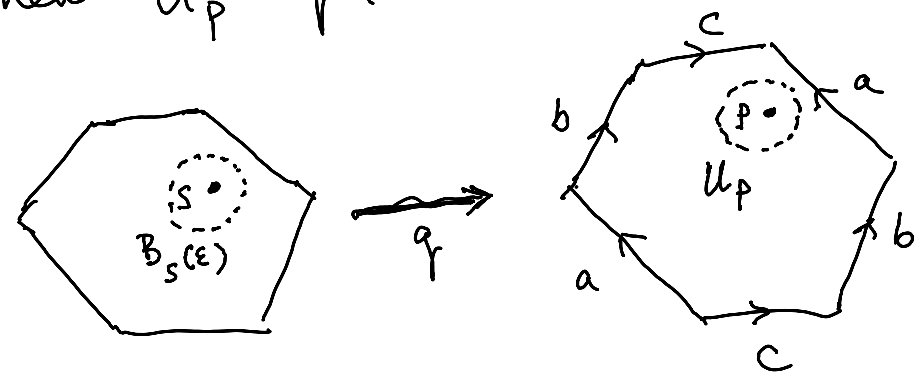
Case 1: if $p = q(s)$ where s is inside the hexagon (i.e. s is not on any edge of it), then let $\varepsilon > 0$ be s.t.

$B_s(\varepsilon) \subset$ the hexagon

(i.e. $B_s(\varepsilon)$ does not intersect any edges)

then $U_p = q(B_s(\varepsilon))$ should work.



Case 2:

if $p = q(s)$ where s is on an edge of the hexagon, but s is not a vertex, then $\exists ! s'$ s.t.

$q(s) = q(s')$, since edges are identified in pairs.

Consider s and s' and let $\varepsilon > 0$ be s.t. $B_s(\varepsilon)$, $B_{s'}(\varepsilon)$ do not contain any vertices and are disjoint.

Then the half-discs

$B_s(\varepsilon) \cap \text{hexagon}$ and $B_{s'}(\varepsilon) \cap \text{hexagon}$

"glue to a disc" i.e. we claim that

$$U_p = q\big(B_s(\varepsilon) \cap \text{hexagon}\big) \cup q\big(B_{s'}(\varepsilon) \cap \text{hexagon}\big)$$
$$= q\Big(\big(B_s(\varepsilon) \cup B_{s'}(\varepsilon)\big) \cap \text{hexagon}\Big)$$

will work.