(b) i) Suppose $\{V_\beta\}_{\beta=1}^{n}$ is a finite collection of closed sets.

Want to show that $\bigcup_{\beta=1}^{n} V_\beta$ is also closed.

Proof: Since V_β is closed for $\beta = 1, \dots, n$, the sets $X \setminus V_\beta$ are open in X. Thus $\bigcap_{\beta=1}^{n} (X \setminus V_\beta)$ is open (by axiom 2 of topologies).

By de Morgan Laws: $\bigcap_{\beta=1}^{n}(X \setminus V_\beta) = X \setminus \bigcup_{\beta=1}^{n} V_\beta$

which is then an open set.

So its complement, $\bigcup_{\beta=1}^{n} V_\beta$ is a closed set.

(ii) Now suppose $\{V_\beta\}_\beta$ is an arbitrary collection of closed sets.

~~So~~ Want to show $\bigcap_\beta V_\beta$ is a closed set.

Proof: Since V_β is closed $\forall \beta$, the complements $X \setminus V_\beta$ are open $\forall \beta$.

Thus $\bigcup_\beta (X \setminus V_\beta)$ is an open set, (by axiom (3) of topologies).

Now, by the de Morgan Laws

$$\bigcup_\beta (X \setminus V_\beta) = X \setminus \bigcap_\beta V_\beta,$$ so the set on the right-hand-side is also open.

Thus its complement, $\bigcap_\beta V_\beta$ is closed.

c) E.g. If $U_n = [\frac{1}{n}, 1]$, then $\bigcup_{n=1}^{\infty} U_n = (0,1]$ is neither closed nor open in X. $(n \in \mathbb{N})$

If $U_n = [0,1]$ $\forall n$, then $\bigcup_{n=1}^{\infty} U_n = [0,1]$ is closed. For $U_n = [\frac{1}{n}, +\infty)$

$\bigcup_{n=1}^{\infty} U_n = (0, +\infty)$ is open.

Scanned by CamScanner