(a) f is continuous $\iff$ $\forall V \subset Y$, closed in Y
we have $f^{-1}(V)$ closed in X.

Pf: Note that since by def
$$f^{-1}(V) = \{x \in X \mid f(x) \in V\}$$
we have
$$f^{-1}(Y \setminus V) = X \setminus f^{-1}(V).$$
(This is just "set-theory" i.e. true $\forall f: X \to Y$.)

Now, if $V \subset Y$ is closed then $Y \setminus V \in \tau_Y$

Since f is continuous,

$f^{-1}(Y \setminus V) \in \tau_Y$. But $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V)$ so

$f^{-1}(V)$ is a closed set & we are done.

If f ~~is contin~~ satisfies the property on the right-hand side, then $\forall V \in \tau_Y$, we have $Y \setminus V$ closed in Y, so $f^{-1}(Y \setminus V)$ closed in X.
But $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V)$, so $f^{-1}(V)$ is open in X and we are done, since we have $f^{-1}(V)$ is open, if V is open, which means f is continuous, by definition.