(b) I a.) f continuous $\not\Rightarrow$ f open.

E.g. $f: \mathbb{R} \to \mathbb{R}$, $f(x) = 1$ (constant map) takes the open set $(0,1)$ onto $\{1\}$ a non-open set.

But f is continuous.

f open $\not\Rightarrow$ f continuous.

E.g. $f: X = \mathbb{R} \to Y = \mathbb{R}$ and $f(x) = x$ with τ_X = anti-discrete, τ_Y = discrete is clearly open but not continuous.

(Actually, taking τ_X = discrete & τ_Y = anti-discrete is another example of a continuous but not open map.

(b) II. continuous $\not\Rightarrow$ this property

e.g. take $f: \mathbb{R}^2 \to \mathbb{R}$
$(x,y) \mapsto x$ projection.

Clearly, continuous (a polynomial).

For $U = (1,2) \times \{3\}$ we have $f(U) = (1,2) \subset \mathbb{R}$ which is open in $\mathbb{R}$, but U is not open in $\mathbb{R}^2$.

This property $\not\Rightarrow$ continuous

e.g. take $f: \mathbb{R} \to \mathbb{R}$ with $f(x) = \begin{cases} 1 & \text{if } x \leq 0 \\ 2 & \text{if } x > 0. \end{cases}$

f is clearly not continuous. But f satisfies this property (vacuously) since $\forall U \subset \mathbb{R}$ we have $f(U) = \{1\}$ or $\{2\}$ or $\{1,2\}$ which are ~~never~~ not open in $\mathbb{R}$.

(NOTE: $\mathbb{R}$, $\mathbb{R}^2$ are considered with the usual topology)