Preserver Weekend in Szeged

Bolyai Institute
University of Szeged, Szeged, Hungary
12-14 April 2019

Book of abstracts

The workshop is supported by Ministry of Human Capacities, Hungary
grant no. 20391-3/2018/FEKUSTRAT
and
National Research, Development and Innovation Office – NKFIH, grant no. K115383.

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16:35–16:50 M. Cueto-Avellaneda
Extension of isometries on the unit sphere of some spaces of continuous functions

17:00–17:25 Gy. P. Gehér
Isometric embeddings of Wasserstein spaces

17:25–17:50 *Coffee break*

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Linear maps preserving invariants arising from combinatorial matrix theory

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Matrix majorizations and their preservers

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Yet another preserver of (local) compactness – maps respecting the compatibility ordering

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Preservers on the Schur product

17:35–17:50 P. Szokol
Preserving problems related to different means of positive operators

Sunday morning session. Chair: Y.-F. Lin

09:00–09:25 F. J. Fernández-Polo
Scattered C^*-algebras and relatively weakly convex combinations of slices

09:35–10:00 L. Arambašić
On Roberts orthogonality in C^*-algebras

10:10–10:35 M. Pálfia
Sturm's law of large numbers for the L^1-Karcher mean of positive operators

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11:05–11:10 Closing

11:30–13:00 *Lunch*

On Roberts orthogonality in C^*-algebras

Ljiljana Arambašić
University of Zagreb

We say that the elements x and y of a complex normed space X are Roberts orthogonal if $\|x + \lambda y\| = \|x - \lambda y\|$ for all complex numbers λ. Let A be a unital C^*-algebra with the unit e. In this talk we present results which characterize Roberts orthogonality of the unit e and an arbitrary element a of A. The talk is based on joint work with T. Berić and R. Rajić. This research has been fully supported by the Croatian Science Foundation under the project IP-2016-06-1046.

Spectral order isomorphisms

Martin Bohata
Czech Technical University in Prague

The spectral order is a partial order on self-adjoint operators. In contrast with the standard operator order, the self-adjoint part of an AW*-algebra endowed with the spectral order forms conditionally complete lattice (i.e. a lattice in which every nonempty bounded subset has the infimum and the supremum). By a spectral order isomorphism we mean a bijection between subsets of self-adjoint parts of AW*-algebras which preserves the spectral order in both directions. In this talk, we investigate conditions ensuring that spectral order isomorphisms are given by a composition of functional calculus with a suitable map induced by an isomorphism of projection lattices.

Zero Product Determined Algebras

Matej Brešar
University of Ljubljana and University of Maribor

An algebra A over a field F is said to be zero product determined if for every bilinear map $f : A \times A \to F$ with the property that $ab = 0$ implies $f(a, b) = 0$, there exists a linear functional φ on A such that $f(a, b) = \varphi(ab)$ for all $a, b \in A$. In the context of Banach algebras, one adds the assumption that f and φ are continuous. The talk will give a brief survey on examples and properties of these algebras.

Linear maps preserving matrices of local spectral radius zero

Constantin Costara
Ovidius University of Constanţa

In this talk, we shall give a characterization of linear maps on matrix spaces which preserve matrices of local spectral radius zero at some fixed nonzero vector. This is a joint work with Abdellatif Bourhim.

Extension of isometries on the unit sphere of some spaces of continuous functions

María Cueto-Avellaneda
University of Granada

In 1987 D. Tingley found inspiration in the celebrated Mazur-Ulam theorem, together with the Mankiewicz's generalization, to ask if every surjective isometry between the unit spheres of two normed spaces is necessarily the restriction of a surjective real linear isometry between the whole spaces.

Tingley's problem motivated L. Cheng and Y. Dong to introduce in 2011 the *Mazur–Ulam property*: a Banach space X satisfies the Mazur–Ulam property if for any Banach space Y, every surjective isometry $\Delta : S(X) \to S(Y)$ admits an extension to a surjective real linear isometry from X onto Y, where $S(X)$ and $S(Y)$ denote the unit spheres of X and Y, respectively. Actually, an equivalent reformulation tells that X satisfies the Mazur–Ulam property if Tingley's problem admits a positive solution for every surjective isometry from $S(X)$ onto the unit sphere of any Banach space Y.

This talk is devoted to present a new example of a Banach space satisfying the Mazur–Ulam property. Concretely, let K be a compact Hausdorff space and let H be a real or complex Hilbert space with $\dim(H_{\mathbb{R}}) \geq 2$. We shall show that the space $C(K,H)$ of all H-valued continuous functions on K, equipped with the supremum norm, satisfies the Mazur–Ulam property. We shall also make a brief incursion into the structure of $C(K)$-module of $C(K,H)$ as well as into several useful results in JB*-triple theory on which our strategy relies.

Scattered C*-algebras and relatively weakly convex combinations of slices

F.J. Fernandez-Polo
University of Granada

Quite recently it was shown that a compact Hausdorff topological space, K, is scattered if and only if the space $C(K)$ of continuous functions on K satisfies that every convex combination of slices of the closed unit ball is relatively weakly open. This can be considered as a new characterization of scatteredness in the case of commutative C*-algebras. We study this property in the setting of general C*-algebras.

We prove, among other results, that given a compact C*-algebra, $\mathcal{A}$, $C(K, \mathcal{A})$ satisfies that every convex combination of slices of the closed unit ball is relatively weakly open if and only if K is scattered and $\mathcal{A}$ is the c_0-sum of finite-dimensional C*-algebras.

Isometric embeddings of Wasserstein spaces

György Pál Gehér
University of Reading

If we consider a metric space (X, d) it is always a natural problem to ask how can we describe the semi-group $\mathrm{IsEmb}(X)$ of all *isometric embeddings* (distance preserving self-maps), and the group $\mathrm{Isom}(X)$ of all *isometries* (bijective distance preserving self-maps). There are several classical results in functional analysis that does this for linear isometries and linear isometric embeddings, and even today intensive research is going on in this direction. For instance, the famous *Banach–Stone* theorem characterises all linear isometries of the Banach space $C(K)$ of all continuous functions on a compact Hausdorff space K. Another famous example is the *Banach–Lamperti* theorem, which characterises all linear isometric embeddings of the Banach space $L^p(\Omega, \mathcal{A}, \mu)$ where $(\Omega, \mathcal{A}, \mu)$ is a σ-finite measure space and $0 < p < \infty$.

In the last two decades, there has been a great interest in characterising the groups of isometries of certain *metric spaces of Borel probability measures*.

This has been done for many cases, including the Kolmogorov-Smirnov, Levy, Levy-Prokhorov, and Kuiper metrics. The importance of these distances lies in the fact that they metrise the weak-* convergence of probability measures. The well-known *p-Wasserstein distance* $(0 < p < \infty)$ is probably the most often used metric in many areas of mathematics and its applications, furthermore, it also metrises the weak-* convergence of measures. Recently, the isometries of 2-Wasserstein spaces have been studied extensively by Bertrand and Kloeckner in several papers. We would like to emphasise that in those papers both the choice of the specific parameter $p = 2$ and the assumption of bijectivity were crucial.

Recently we became interested in describing the semi-group of all isometric embeddings of p-Wasserstein spaces for a general parameter p. The talk will be an overview of our recent results.

Joint work with Tamás Titkos (MTA Renyi Institute, Budapest) and Dániel Virosztek (IST Austria, Klosterneuburg).

LINEAR MAPS PRESERVING INVARIANTS ARISING FROM COMBINATORIAL MATRIX THEORY

Alexander Guterman
Lomonosov Moscow State University

The theory of transformations preserving different matrix properties and invariants dates back to the works by Frobenius, Schur, and Dieudonné, see [1, 5, 2] and is an intensively developing part of linear algebra and its applications nowadays. The detailed and self-contained exposition on the topic can be found in a number of good surveys and monographs, see for example [4].

It is an actual subject to investigate these transformations for combinatorial or graph theory invariants. In the talk we present our recent results on this subject. We plan to characterize the maps preserving cyclicity index of graphs, scrambling index and its generalizations, primitivity index for graphs and k-colored graphs, tournament graphs, etc.

[1] G. FROBENIUS, Über die Darstellung der endlichen Gruppen durch lineare Substitutionen, Sitzungsber., Preuss. Akad. Wiss (Berlin), Berlin, 1897, pp. 994-1015.

[2] J. DIEUDONNÉ, Sur une généralisation du groupe orthogonal à quatre variables, *Arch. Math.* **1** (1949), 282–287.

[3] G. PÓLYA, Aufgabe, **424**, *Arch. Math. Phys.* **20**, 3 (1913), 271.

[4] S. PIERCE AND OTHERS, A survey of linear preserver problems, *Linear and Multilinear Algebra* **33** (1992), 1–119.

[5] I. SCHUR, *Einige Bemerkungen zur Determinantentheorie*, Akad. Wiss. Berlin: S.-Ber. Preuß, 1925, 454–463.

MORPHISMS OF VON NEUMANN FACTORS WELL BEHAVED WITH RESPECT TO COMMUTING ELEMENTS

Jan Hamhalter
Czech Technical University in Prague

Let G_1 and G_2 be sets endowed with binary operations $*_1$ and $*_2$, respectively. A map $\varphi : G_1 \to G_2$ is called a piecewise homomorphism if

$$\varphi(a *_1 b) = \varphi(a) *_2 \varphi(b) \quad \text{whenever} \quad a *_1 b = b *_1 a\,.$$

If φ is a bijection such that both φ and φ^{-1} are piecewise homomorphisms, then φ is called a piecewise isomorphism. Piecewise morphisms between C^*-algebras were introduced by Heunen and Ryes in connection with mathematical foundations of quantum theory. The aim of this talk is to present a complete description of piecewise isomorphisms in the following cases:

- Let A_+^{-1} denote the group of positive invertible elements of a unital C* algebra A (with respect to a usual multiplication). Let $\varphi : N_+^{-1} \to M_+^{-1}$ be a weak* continuous piecewise isomorphism, where M and N are von Neumann factors with $\dim N \geq 9$. Then φ has the following form:

$$\varphi(a) = e^{\psi(\log a)}\, \theta(a^c)\,,$$

where ψ is a bounded linear functional on M, c a nonzero real number, and $\theta : M \to N$ is either a $*$-isomorhism or a $*$-antiisomorhism.

- Let A_u denote the unitary group of a unital C* algebra A (with respect to a usual multiplication). Let $\varphi : N_u \to M_u$ be a bicontinuous piecewise isomorphism, where M and N are von Neumann factors with $\dim N \geq 9$. Then φ has the following form:

$$\varphi(u) = e^{i\psi(-i \log u)}\, \theta(u^c)\,,$$

where ψ is a bounded hermitian linear functional on N, c a nonzero real number, and $\theta : M \to N$ is either a $*$-isomorhism or a $*$-antiisomorhism.

Piecewise morphisms between general C*-algebras and their connection with the problem of preserving commutativity will also be discussed.

[1] J.Hamhalter Piecewise *-homomorphisms and Jordan maps on C*-algebras and factor von Neumann algebras, *J. Math.Anal.Appl. 462 (2018), 1014-1031.*

Perturbations of homomorphisms of Banach algebras

Bence Horváth
Institute of Mathematics of the Czech Academy of Sciences

Perturbations of characters of Banach algebras were first studied by Jarosz in [2]. His work was substantially extended by Johnson in [3] and [4], where the target space $\mathbb{C}$ is replaced with an arbitrary Banach algebra B. In this latter paper Johnson defines the so-called *AMNM property (Almost Multiplicative maps are Near a Multiplicative)* for a pair of Banach algebras (A, B). Roughly speaking, this property is concerned with the following question: Let A, B be Banach algebras. For a bounded linear map $\phi : A \to B$ let us define the *multiplicative defect* of ϕ as

$$\operatorname{def}(\phi) := \sup\{\|\phi(ab) - \phi(a)\phi(b)\| : \ a, b \in A, \|a\|, \|b\| \leq 1\}.$$

Let $\operatorname{Mult}(A, B)$ denote the (closed) set of continuous algebra homomorphisms from A to B. For a bounded linear map $\phi : A \to B$, how does the quantity $\operatorname{def}(\phi)$ relate to $\operatorname{dist}(\phi, \operatorname{Mult}(A, B))$? In our talk we shall investigate this question with special emphasis on algebras of operators on Banach spaces: In [4] it was shown that $(\mathcal{B}(\mathcal{H}), \mathcal{B}(\mathcal{H}))$ has the AMNM property, where $\mathcal{H}$ is a separable Hilbert space. The present author recently extended this result in [1] with Y. Choi and N. J. Laustsen (Lancaster) to a certain class of non-Hilbertian Banach spaces. Although the study of the AMNM property has an intimate connection with the cohomology of Banach algebras ([5]), we note that *almost multiplicative* and *almost additive* maps played a hugely important role in Farah's work in proving that all automorphisms of the Calkin algebra $\mathcal{C}(\mathcal{H})$ are inner, assuming Todorcevic's Axiom [6].

[1] Y. Choi, B. Horváth, and N. J. Laustsen. Johnson's AMNM property for endomorphisms of $\mathcal{B}(E)$. *In preparation.*

[2] K. Jarosz. *Perturbations of Banach algebras*, Springer-Verlag, Berlin, 1985.

[3] B. E. Johnson. Approximately multiplicative functionals. *J. Lond. Math. Soc.*, 34(3):489–510, 1986.

[4] B. E. Johnson. Approximately multiplicative maps between Banach algebras. *J. Lond. Math. Soc.*, 37(2):294–316, 1988.

[5] B. E. Johnson. Cohomology in Banach algebras. *Memoirs of the AMS.*, 127, 1972.

[6] I. Farah. All automorphisms of the Calkin algebra are inner. *Annals of Mathematics*, 173:619–661 2011.

Surjective linear isometries with finite spectrum

Dijana Ilišević
University of Zagreb

Let $\mathcal{X}$ be a complex Banach space and let $T\colon \mathcal{X} \to \mathcal{X}$ be a surjective linear isometry with the spectrum $\sigma(T) = \{\lambda_1, \ldots, \lambda_n\}$. Every λ_i is an eigenvalue of T and if P_i is the projection onto the kernel of $T - \lambda_i I$ (the so-called eigenprojection) then

$$P_1 \oplus \cdots \oplus P_n = I \quad \text{and} \quad \lambda_1 P_1 + \lambda_2 P_2 + \cdots + \lambda_n P_n = T.$$

An important class of surjective linear isometries with finite spectrum is the class of periodic linear isometries: T is periodic of period m if $T^m = I$ and $T^k \neq I$ for $k = 1, \ldots, m-1$.

Eigenvalues and eigenprojections of surjective linear isometries on a large class of complex Banach spaces will be discussed. A particular emphasis will be given to $C_0(\Omega)$, the Banach space of continuous complex-valued functions on a locally compact Hausdorff space Ω vanishing at infinity.

The recent results that will be presented in this talk are from joint work with Fernanda Botelho (University of Memphis, USA) and joint work with Chih-Neng Liu and Ngai-Ching Wong (National Sun Yat-sen University, Taiwan). The work of Dijana Ilišević has been fully supported by the Croatian Science Foundation under the project IP-2016-06-1046.

Yet another preserver of (local) compactness –maps respecting the compatibility ordering

Tomasz Kania
Czech Academy of Sciences in Prague

Let f and g be two scalar-valued continuous functions on a topological space. We say that g dominates f in the compatibility ordering whenever g agrees with f on the support of f. The aim of the talk will be to report on a recent result with M. Rmoutil, which roughly asserts that a compact, Hausdorff space may recovered from this ordering. We will then derive the classical preserver theorems such as the Gelfand–Kolmogorov theorem for ring/algebra isomorphisms, the Banach–Stone theorem for isometries, Arens' theorem for Banach-lattice isomorphisms, and Kaplansky's theorem for pointwise-ordering isomorphisms as easy corollaries.

On diameters of commuting graphs

Bojan Kuzma
University of Primorska

A commuting graph was the main tool we used to classify surjective (possibly nonlinear) maps which in one direction only preserve commutativity on complex/real matrices. It also brought new insights into the importance of the commutativity relation. Namely algebraic sets within certain categories are determined up to isomorphism by commutativity relation alone. Examples of such categories are finite simple groups or Banach algebras of bounded operators on complex Hilbert spaces.

In the talk we will present some of our recent results into the basic question of commuting graphs – what is its diameter. The answer to this question is fundamental in classification of (possibly nonlinear) commutativity preservers.

This is a joint work with G. Dolinar, J. Marovt, P. Oblak, D. Kokol-Bukovšek, and D. Dolžan.

Preservers on the Schur product

Ying-Fen Lin
Queen's University Belfast

Let $a = (a_{ij})$ and $b = (b_{ij})$ be two $m \times n$ matrices. The *Schur product* of a and b is given by the entry-wise product $(a_{ij}b_{ij})$. Such a product can still be defined on the infinite dimensional matrix space, this is, the Banach space $B(\ell^2)$ of all bounded linear operators on ℓ^2, and beyond. In this talk, I am going to present some characterisations of linear preservers associated with the Schur product. To be precise, we describe the Schur multiplicative and Schur-null preserving maps on both finite and infinite dimensional matrix spaces, and some isometries with respect to the Schur product.

Rank-one nonincreasing maps on symmetric matrices over small fields

Marko Orel
University of Primorska & IMFM

Let $S_n(\mathbb{F})$ be the set of all $n \times n$ symmetric matrices over a field $\mathbb{F}$. An additive map $\Phi : S_n(\mathbb{F}) \to S_n(\mathbb{F})$ is *rank-one nonincreasing* if it maps matrices of rank one to matrices of rank at most one. The characterization of such maps dates back to 2005/2006 [1, 2] in the case the field $\mathbb{F}$ has at least four elements. Moreover, in [2] it was proved that if such map Φ has in its image a matrix of rank ≥ 4, then it is necessarily of the standard form $\Phi(A) = aTA^\sigma T^\top$, regardless of the field $\mathbb{F}$. Here, a, T and σ are a fixed scalar, a fixed matrix and a fixed field endomorphism, respectively. About a decade ago, it seemed that the characterization of all rank-one nonincreasing maps in the case $|\mathbb{F}| \in \{2, 3\}$ is not feasible. However, recently we were able to understand the nature of such maps and provide their full characterization [3]. The key tool was a better understanding of the union of certain (hyperbolic/parabolic) quadrics. This additional knowledge helped us to realize that the two smallest fields are not so special after all. In fact, the same kind of nonstandard maps appear in the case of larger fields if one studies analogous (open) problem on symmetric tensors.

[1] M-H. Lim. Rank-one nonincreasing additive mappings on second symmetric product spaces, Linear Algebra Appl., 402:263–271, 2005.

[2] B. Kuzma and M. Orel. Additive mappings on symmetric matrices, Linear Algebra Appl., 418:277–291, 2006.

[3] M. Orel. Nonstandard rank-one nonincreasing maps on symmetric matrices, Linear and Multilinear Algebra, 67(2):391–432, 2019.

STURM'S LAW OF LARGE NUMBERS FOR THE L^1-KARCHER MEAN OF POSITIVE OPERATORS

Miklós Pálfia
University of Szeged / Sungkyunkwan University

Firstly we briefly review some available versions of the strong law of large numbers in Banach spaces and nonlinear extensions provided by Sturm in CAT(0) metric spaces. Sturm's 2001 L^2-result was directly applied to the case of the geometric (also called Karcher) mean of positive matrices, thus it suggests a natural formulation of the law for positive operators. However there are serious obstacles to overcome to prove the law in the infinite dimensional case. We propose to use a recently established gradient flow theory by Lim-P for the Karcher mean of positive operators and a stochastic proximal point approximation to prove the L^1-strong law of large numbers for the Karcher mean in the operator case.

PRESERVERS OF λ-ALUTHGE TRANSFORMS ON PRODUCTS

Antonio M. Peralta
University of Granada

Let a be an element in a von Neumann algebra M. Let $a = u|a|$ be the polar decomposition of a in M, where u is a partial isometry in M, $|a| = (a^*a)^{\frac{1}{2}}$, and u^*u is the range projection of $|a|$. Given $\lambda \in [0, 1]$, the λ-*Aluthge transform* of a is defined by $\Delta_\lambda(a) = |a|^\lambda u|a|^{1-\lambda}$. Suppose N is another von Neumann algebra.

Continuing with the topic of a talk by M. Mbekhta in the meeting "*Preservers Everywhere, Szeged'2017*", we shall present in this talk some new results in the study of those bijective maps between von Neumann algebras commuting with the λ-Aluthge transform on products of the form ab, ab^*, $a \circ b$ and $a \circ b^*$, where $\circ$ denotes the natural Jordan product. We shall show that all these maps are naturally linked to the Jordan structure of the von Neumann algebras. We shall also see how these new results are connected with the classical studies by J. Hakeda and K. Saitô on linear bijections between von Neumann algebras preserving products of the form ab and $a \circ b$.

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Endomorphisms of the poset of idempotent matrices

Peter Šemrl
University of Ljubljana

Let D be a division ring and $P_n(D)$ the set of all $n \times n$ idempotent matrices over D. There is a natural partial order on $P_n(D)$ defined by $P \leq Q$ if and only if $PQ = QP = P$. The general form of the authomorphisms of this poset was obtained by Ovchinnikov in 1993. We will discuss possible improvements of his result. Automorphisms are bijective maps preserving order in both directions. Can we relax the bijectivity assumption? Can we obtain a similar result under assuming that order is preserved in one direction only? These questions were motivated by our study of the optimal version of Hua's fundamental theorem of geometry of matrices. Recently we have obtained definitive results in this direction.

Matrix majorizations and their preservers

Pavel Shteyner
Lomonosov Moscow State University

Joint work with G. Dahl and A. Guterman. Based on results published in [1].

The notion of majorization is known since 1903 and dates back to the works by Muirhead, Lorenz, Dalton, Schur, Hardy, Littlewood and Pólya.

For a vector $x \in \mathbb{R}^n$ we let $x_{[j]}$ denote the jth largest number among the components of x. We say that y is *majorized* by x, ($y \preceq x$), if

$$\sum_{j=1}^{k} y_{[j]} \leq \sum_{j=1}^{k} x_{[j]} \quad k = 1, \ldots, n-1 \text{ and } \sum_{j=1}^{n} y_j = \sum_{j=1}^{n} x_j$$

Majorization is useful in abstract mathematics as well as in applications such as combinatorial analysis, numerical analysis, statistics, probability theory, economics, physics and chemistry, etc.

The classical concept of vector majorization $y \preceq x$ can be extended to matrix space $M_{n,m}$ in many ways. For example:

- Strong majorization: $A \preceq^s B$ when there is $X \in \Omega_m$ such that $A = XB$.
- Directional majorization: $A \preceq^d B$ when $Ax \preceq Bx$ for all $x \in \mathbb{R}^m$.
- Weak majorization: $A \preceq^w B$ when there is $X \in \Omega_m^{row}$ such that $A = XB$.

Here Ω_m (Ω_m^{row}) is the set of all doubly (row) stochastic matrices of order m.

Majorizations of matrices were extensively studied and some of their applications, for example theory of statistical experiments required a further generalization. We propose the following notion.

Let $\mathcal{A}$ and $\mathcal{B}$ be two, possibly, finite classes of matrices in $M_{n,m}$. We say that $\mathcal{A}$ is *majorized* by $\mathcal{B}(\mathcal{A} \trianglelefteq \mathcal{B})$ if for all $A \in \mathcal{A}$ there exists $B \in \mathcal{B}$ such that $A \preceq B$.

$$\mathcal{A} \trianglelefteq \mathcal{B} \Rightarrow \forall A \in \mathcal{A} \ \exists B \in \mathcal{B} : A \preceq B$$

A natural question is to find, for a given matrix class $\mathcal{A}$, a matrix class $\mathcal{B} = \operatorname{argmin}\{|\mathcal{C}| : \mathcal{A} \trianglelefteq \mathcal{C}\}$. For example, for directional majorization we have the following:

Theorem 1. *Let $\mathcal{A}$ be a matrix class in $M_{n,m}$. Assume there exists a certain matrix $B \in M_{n,m}$ such that $\mathcal{A} \trianglelefteq^w \{B\}$ and the vectors of column sums of all $A_i \in \mathcal{A}$ coincide, that is $e^t A_i = e^t A_j$ for all i, j. Then there exists $C \in M_{n,m}$ such that $\mathcal{A} \trianglelefteq^d \{C\}$.*

Another research direction addressed in the talk is the theory of linear maps preserving/converting matrix majorizations. In particular for maps converting majorization we have the following result:

Theorem 2. *Let T be a linear operator on $M_{n,m}$. The following are equivalent:*

1. *$T(A) \preceq^s T(B)$ whenever $A \preceq^d B$ for any $A, B \in M_{n,m}$*
2. *One of the following hold:*
   (a) *There exist $S_1, \ldots, S_m \in M_{n,m}$ such that $T(X) = \sum_{j=1}^{m} (e^t x^j) S_j$.*
   (b) *There exist $S \in M_m$, $P \in S_n$ and $R \in M_m$ with $A \preceq^d B \Rightarrow AR \preceq^s BR$, such that $T(X) = PXR + JXS$.*

[1] G. Dahl, A. Guterman, P. Shteyner. Majorization for matrix classes. Linear Algebra Appl., 2018. No. 555. pp. 201–221.

Preserving problems related to different means of positive operators

Patricia Szokol
University of Debrecen

In this presentation, we mainly discuss the problem of describing the structure of transformations leaving norms of generalized weighted quasi-arithmetic means of invertible positive operators invariant. Under certain conditions, we present the solution of this problem which generalizes one of our former results containing its solution for weighted quasi-arithmetic means. Moreover, we investigate the relation between the generalized weighted quasi-arithmetic means and the Kubo-Ando means.

Maps preserving absolute continuity of positive operators

Zsigmond Tarcsay
Eötvös Loránd University, Budapest

Let H be a complex Hilbert space and denote by $B(H)_+$ the set of all positive operators on H. We say that $A \in B(H)_+$ is absolutely continuous with respect to $B \in B(H)_+$ if, for every sequence x_n in H, $(A(x_n - x_m), x_n - x_m) \to 0$ and $(Bx_n, x_n) \to 0$ imply $(Ax_n, x_n) \to 0$. The aim of this talk is to describe the general form of those bijective maps $\phi : B(H)_+ \to B(H)_+$ which preserve absolute continuity in both direction.