Tingley's problem for subsets strictly smaller than the unit sphere

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The celebrated Tingley's problem has focused the attention of a wide community of researchers on preservers in recent years. It admits the following easy statement: Suppose $\Delta : S(X) \to S(Y)$ is a surjective isometry between the unit spheres of two Banach spaces X and Y. Does Δ admit an extension to a surjective linear isometry from X onto Y? This difficult problem remains open even in the case of 2-dimensional spaces. A long series of papers has been devoted to provide positive answers for some concrete structures, these partial answers have produced a wide range of new tools and results with interesting geometric and analytic conclusion.

The reader might guess from the title that we won't limit ourself to Tingley's problem in this talk. It is natural to challenge the audience to consider other variants. We shall deal with one of the most attractive and we shall consider the posibility of extending surjective isometries between proper subsets of the unit spheres (for example, the subset of extreme points of the closed unit ball, the subset of positive elements in the unit sphere of $B(H)$, the subgroup of unitary elements in a unital C*-algebra, the set of unitary elements in a unital JB*-algebra, etcetera). We shall see that negative and positive answers can be obtained.

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